Abstract—State of the art spectrum sensing methods evaluate sensing performance only based on correct detection of the presence of the primary user’s signal. As a first step, we propose a more realistic framework that also takes the correct detection of the absence of primary user’s signal into account for spectrum sensing performance evaluation. More precisely, by using the above framework, we select energy detection spectrum sensing parameters so as to minimize a cost function leading to an optimal operating point over the widely-used receiver operating characteristic curve. In a second step, by using the parameters obtained in the first step, we calculate the information rates achieved by both the cognitive and the primary users. These rates are compared to those provided by an ideal (interference-free) spectrum sensing. Numerical results show that the proposed improved spectrum sensing outperforms conventional spectrum sensing techniques in terms of total achievable throughputs, without any increase in the cognitive radio complexity.

I. INTRODUCTION

Cognitive radio (CR) technology [1] has been recommended as a key solution to the problem of inefficient use of allocated spectrum to primary licensed users. CR allows unlicensed or secondary users (SU) to access spectrum bands which has been allocated to licensed or primary users (PU) when interference to PU remains below a given threshold. To reach this goal, CR users have to sense the spectrum constantly in order to detect the presence of a primary transmitter signal (PTS). Therefore, spectrum sensing is one of the most important issues in each CR network. Due to channel fading conditions and the so-called hidden terminal problem [2], spectrum sensing is usually imperfect and imposes interference on the primary network. Cooperative spectrum sensing introduced, for instance, in [2], [3], [4], [5], [6] and in [7] improves the detection reliability by employing multiple cognitive users, i.e., by exploiting the available spatial diversity.

One of the widely-used techniques for spectrum sensing is energy detection (ED) [3], [4], [8]. In this technique, each CR senses the spectrum during a sensing period and measures the received energy over a particular spectrum band. The measured energy is then compared to a threshold value and a binary hypothesis is made about the presence or the absence of the primary user. Spectrum sensing performance evaluation is usually done based on the so-called receiver operating characteristic (ROC) curves. These ROC curves plots the probability of miss-detection \( P_m \) (the probability that the CR fails to detect the presence of the PU) versus the probability of false-alarm \( P_f \) (the probability that the CR decides the PU is in operation whereas it was actually off). In conventional ED techniques, the threshold value is set so as to satisfy a maximum false-alarm probability. A capacity analysis for the cognitive channel is provided in [9]. However, in [9], the interference caused by primary signal on the cognitive signal and its relation to spectrum sensing methods and parameters are not taken into account.

In this paper, we show that the conventional method for selecting the threshold value leads to minimizing the misdetection, or equivalently maximizing the false-alarm probability. Furthermore, we introduce new probability metrics for setting the threshold value in ED and show that the conventional method can be viewed as a special case of our general framework. Then we provide the expression of the achievable information rates associated to a cognitive system based on the improved and the conventional ED spectrum sensing, by taking into account the interference from the primary network. We also derive the total sum-rate of the primary and cognitive networks. Our results may serve in the evaluation of the trade-off between the required achieved throughputs and CR spectrum sensing parameters (i.e., misdetection and false-alarm probabilities).

The rest of this paper is organized as follows. In Section II we introduce our new spectrum sensing performance metrics by taking the absence of the PU into account. In Section III, we explain our spectrum sensing system model. We also formulate our improved spectrum sensing parameter selection as an optimization problem based on the metrics defined in Section II. In Section IV, we calculate the achievable rates for both the primary and the cognitive network based on the improved and the conventional ED spectrum sensing. Section V provides simulation results and discussions about the performance of the proposed technique. Finally, Section VI draws our conclusions.

II. A NEW PROBABILISTIC FRAMEWORK FOR SPECTRUM SENSING

Sensing the presence of a primary transmitter inside a given frequency band is usually viewed as a binary hypothesis testing
problem with hypothesis $H_0$ and $H_1$ defined as:

$$
\begin{cases}
H_1 & \text{primary user is in operation} \\
H_0 & \text{primary user is not in operation.}
\end{cases} \quad (1)
$$

Obviously, in the above definition, one has to differentiate between the presence (or the absence) of the primary user in reality and from the cognitive radio point of view, i.e., the decision made by spectrum sensing. State of the art contributions such as [3] uses the above two hypotheses to define the following conditional probabilities:

$$
P_m = P(H_0^{CN}|H_1^{PN}) \quad (2)
$$

and

$$
P_f = P(H_0^{CN}|H_0^{PN}) \quad (3)
$$

where $H_0^{PN}$ denotes the absence (for $i=0$) and the presence (for $i=1$) of the primary signal respectively; $H_0^{CR}$ indicates the decision made based on the received signals during spectrum sensing at cognitive terminals about the absence (for $i=0$) and the presence (for $i=1$) of the primary signal. Another key metric widely-used for spectrum sensing is the detection probability $P_d$ [10] defined as:

$$
P_d = 1 - P_m = P(H_1^{CN}|H_1^{PN}). \quad (4)
$$

Note that by using $P_d$ as defined in (4), the conditional event $(H_1^{CN}|H_1^{PN})$ (i.e., correct detection by the CR of the absence of a primary user) is not taken into account for spectrum sensing metric formulation. In what follows, we propose a more general framework by taking into account both the *presence* and the *absence* of the primary signal. More precisely, we define a modified detection probability denoted by $\hat{P}_d$ as:

$$
\hat{P}_d = P(H_0^{CN},H_0^{PN}) + P(H_1^{CN},H_1^{PN})
= P(H_0^{CN}|H_0^{PN}) \cdot P(H_0^{PN}) + P(H_1^{CN}|H_1^{PN}) \cdot P(H_1^{PN}) \quad (5)
$$

Noting that we have $P(H_0^{PN}) + P(H_1^{PN}) = 1$ and $P(H_0^{CN}) + P(H_1^{CN}) = 1$, we get:

$$
\hat{P}_d = p_0 (1-P_f) + p_1 (1-P_m), \quad (6)
$$

where $p_0 \triangleq P(H_0^{PN})$ and $p_1 \triangleq P(H_1^{PN}) = 1-p_0$ are *a priori* probabilities on the absence and the presence of the primary network, respectively. Since initial CR devices are intended to work over licensed TV bands and spectrum sensing performance evaluation is usually done at steady state regime, one can assume that *a priori* probabilities $p_0$ and $p_1$ are available in (6). Also, we define a modified miss-detection probability as:

$$
\tilde{P}_m = 1 - \hat{P}_d. \quad (7)
$$

Substituting (7) in (6) yields to:

$$
\tilde{P}_m = p_0 P_f + p_1 P_m \quad (8)
$$

Note that the above modified metrics constitute a more general framework since by setting $p_1 = 1$ (or equivalently $p_0 = 0$), the modified probabilities in (6) and (8) become equivalent to the classical definitions of (4) and (2), respectively. In Section III, the operating point over the spectrum sensing ROC will be selected so as to minimize the modified miss-detection probability of (8).

III. IMPROVED SPECTRUM SENSING

Here, we explain our spectrum sensing system model. We assume the widely-used ED spectrum sensing technique where the test statistic is the observed energy summation within a given sensing period as:

$$
\hat{\theta} = \begin{cases}
H_1 & \text{if } E \geq \zeta \\
H_0 & \text{if } E < \zeta
\end{cases} \quad (9)
$$

where $E$ is the observed energy summation on the subband of interest and $\zeta$ is the applied threshold to differentiate between the two hypothesis $H_0$ and $H_1$. We note that the key point in ED spectrum sensing is the selection of the threshold value $\zeta$. Classically, $\zeta$ is chosen to satisfy a target false-alarm probability [11]. Assume that the maximum acceptable probability of false-alarm is equal to $P_f^{max}$ (point A) and the maximum probability of miss-detection the primary system can support is equal to $P_m^{max}$ (point B). The latter corresponds to the maximum interference level that the primary network can support. These two limits indicate the segment AB over the ROC curve as indicated in Fig. 1. The optimal point over this segment (i.e., the best operating point with respect to a given criteria) is denoted by the point O, i.e., $(P_f^{opt}, P_m^{opt})$. In what follows, we formulate our optimization problem for finding the optimal point $P_{opt}$.

We propose here to select $\zeta$ by using the modified metrics derived in Section II. More precisely, we propose a constrained minimization of the modified miss-detection probability $\tilde{P}_m$ introduced in (8) as follows:

$$
\zeta_{opt} = \arg \min_{\zeta} \{ \tilde{P}_m = p_0 P_f + p_1 P_m \} \quad (10)
$$

subject to:

$$
c_1 : P_m \leq P_m^{max} \\
c_2 : P_f \leq P_f^{max},
$$

where $\tilde{P}_m$ is a function of $\zeta$.

To get more insight on the proposed approach, let us set $p_0 = 0$ in (10) which leads to minimizing $P_m$ over the segment AB in Fig. 1. We note that in this case, the optimal
operating point leading to the minimum probability of miss-detection \( P_m \) would be the point A. Actually, this is equivalent to set the threshold value \( \zeta \) associated to the maximum false-alarm probability \( P_f^{\text{max}} \) as in conventional techniques. Thus, our proposed framework for setting \( \zeta \) is more general in the sense that conventional techniques can be derived from (10) by setting \( p_0 = 0 \). The solution \( \zeta_{\text{opt}} \) of the optimization problem (10) defines an operating point \((P_{f_{\text{opt}}}, P_{m_{\text{opt}}})\) over the ROC curve. The solution of the optimization problem (10) will be find by numerical methods in Section V.

IV. OVERALL INFORMATION RATES ACHIEVED BY THE PRIMARY AND THE COGNITIVE NETWORKS

In this section, we assume that the CR has made a decision by means of spectrum sensing on the presence/absence of the primary network. As illustrated in Fig. 2, in this step, a cognitive transmitter establishes a connection with a cognitive receiver, in addition to the primary link. If the spectrum sensing is ideal, the primary and the cognitive transmissions will not interfere with each other. However, in practice, due to imperfect spectrum sensing, interference occurs since the primary and the cognitive network operates over the same frequency band.

First, we calculate the information rates of the primary link, by assuming the cognitive transmission as a source of interference. In a second step, we calculate the achievable information rates associated to a given operating point \((P_f, P_m)\) over the ROC curve. This enables us to derive the information rates achieved by both the improved and conventional energy detectors. Then, we calculate the overall information sum-rates achieved by the primary and the cognitive networks.

A. Achievable Information Rates Of The Primary Network

Let us first calculate the achievable throughputs for the primary network. To this end, we assume that the primary network uses a fraction \( \alpha \) of the available degrees of freedom (DOF). Fig. 3 illustrates a DOF and the fraction \( \alpha \) used the primary network. In our considered scenario, the cognitive network tries to detect the presence of the primary signal in a fraction \( \alpha \) of DOF and the absence of the primary signal in the rest of DOF, i.e., a fraction \( (1-\alpha) \). Since the power constraint is on the average across the DOF, there is no difference whether the partitioning is across frequency or across time. The achievable throughputs for the primary network using a fraction \( \alpha \) of DOF can be written as [12]:

\[
R^{\text{PN}} = \alpha \log \left( 1 + \frac{S^{\text{PN}}}{\alpha N_0^{\text{PN}}} \right) \text{ bits/s/Hz} \tag{11}
\]

where \( S^{\text{PN}} \) is the received power of primary user (equivalent to \( \frac{S^{\text{PN}}}{\alpha} \) joules energy per DOF) and \( N_0^{\text{PN}} \) is the variance of the additive white Gaussian noise (AWGN) at the primary receiver. Note that the information rates in (11) are achieved when the spectrum sensing is ideal.

However, in practice, spectrum sensing is imperfect. More precisely, in a fraction \( (1-\delta) \) of a fraction \( \alpha \) of DOF, the
received signal at the primary receiver interferes with the cognitive signal. In this case, the achievable throughputs for the primary network in fraction $\alpha$ of DOF is written as:

$$R_{PN}^P = R_{1PN}^P + R_{2PN}^P,$$  

(12)

where $R_{PN}^P$ is the information rates achieved in a fraction $\alpha$ of DOF (depicted in Fig. 3 by region 1) and $R_{2PN}^P$ is the information rates achieved in a fraction $\alpha (1-\delta)$ of DOF when the cognitive network interferes with the primary transmission (region 2). We have:

$$R_{1PN}^P = \alpha \delta \log \left( 1 + \frac{S_{PN}^P}{\alpha N_0^P} \right) \ \text{bits/s/Hz},$$  

(13)

and

$$R_{2PN}^P = \alpha (1-\delta) \log \left( 1 + \frac{S_{PN}^P}{(I_{CN}^P + N_0^{PN})} \right) \ \text{bits/s/Hz},$$  

(14)

where $I_{CN}^P$ is the power of the interference due to imperfect spectrum sensing imposed from the cognitive transmitter on the primary receiver. Note that we have assumed an AWGN model for the interference $I_{CN}^P$ in (14).

Let us now relate the parameters $\alpha$ and $\delta$ involved in (13) and (14) to the probabilistic parameters characterizing our spectrum sensing defined in Section II. We assume that our spectrum sensing is characterized by the operating point $(P_T, P_m)$ over the ROC curve. Thus, by using the definitions introduced in Section II, we can state that:

$$\alpha = P(H_{1PN}^P) = p_1,$$

and

$$\delta = (1 - P_m).$$

In addition, the fraction of DOF associated to each region in Fig. 3 is given by:

- region 1: $p_1 (1 - P_m) = p_1 P_d$,
- region 2: $p_1 P_m$,
- region 3: $(1-p_1) (1 - P_f) = p_0 (1 - P_f)$,
- region 4: $(1-p_1) P_f = p_0 P_f$.

We denote by the fraction $\beta$ of DOF, the resources allocated for cognitive data transmission (depicted in Fig. 3 by the concatenation of regions 2 and 3). We have:

$$\beta = (1-\alpha) (1-P_f) + \alpha P_m = p_0 (1-P_f) + p_1 P_m.$$  

(15)

Finally, the achievable throughputs of the primary network per DOF (equation (12)) can be rewritten in an equivalent form as:

$$R_{PN}^P = p_1 (1 - P_m) \log \left( 1 + \frac{S_{PN}^P}{\frac{p_1}{p_1} N_0^{PN}} \right)$$

$$+ p_1 P_m \log \left( 1 + \frac{S_{PN}^P}{\frac{p_1}{p_1} \left( \frac{S_{PN}^P}{\beta N_0^{PN}} \right)} \right) \ \text{bits/s/Hz},$$  

(16)

where $\frac{S_{CN}^P}{\beta N_0^{PN}} = I_{CN}^P$ with $S_{CN}^P$ being the received power of the cognitive user.

B. Achievable Information Rates Of The Cognitive Network

In the following we calculate the achievable throughput for the cognitive network. The achievable throughputs for the cognitive network with ideal spectrum sensing can be written as:

$$R_{CN}^I = (1-\alpha) \log \left( 1 + \frac{S_{CN}^P}{(1-\alpha) N_0^{CN}} \right) \ \text{bits/s/Hz},$$  

(17)

where $S_{CN}^P$ is the received power from the cognitive terminal, leading to $\frac{S_{CN}^P}{N_0^{CN}}$ joules energy per DOF and $N_0^{CN}$ is the variance of the AWGN at the cognitive receiver. Consider now the achievable throughputs for the cognitive network under imperfect spectrum sensing. In this case, the throughputs in a fraction $\beta$ of DOF is given by:

$$R_{CN}^P = R_{2CN}^P + R_{3CN}^P,$$  

(18)

where $R_{2CN}^P$ is the information rates achieved by the CR when the primary user is considered as interference (i.e., region 2 in Fig. 3) and $R_{3CN}^P$ is the achieved throughputs of the CR in a fraction of DOF without any interference from the primary network (i.e., region 3 in Fig. 3). According to the fraction of DOF associated to each region introduced in Subsection IV-A, each part can be written as:

$$R_{2CN}^P = p_1 P_m \log \left( 1 + \frac{S_{CN}^P}{\beta (I_{PN}^P + N_0^{CN})} \right) \ \text{bits/s/Hz},$$  

(19)

and

$$R_{3CN}^P = p_0 (1 - P_f) \log \left( 1 + \frac{S_{CN}^P}{\beta N_0^{CN}} \right) \ \text{bits/s/Hz},$$  

(20)

where $I_{PN}^P$ is the power of the interference imposed from the primary signal at the cognitive receiver. Again, we have assumed an AWGN model for the interference $I_{PN}^P$ in (19). Obviously, per DOF we have:

$$I_{PN}^P = \frac{S_{PN}^P}{\alpha p_1} = \frac{S_{PN}^P}{p_1}.$$  

(21)

Thus, the information rates achieved by the cognitive network per DOF (equation (18)) can be rewritten in an equivalent form as:

$$R_{CN}^P = p_1 P_m \log \left( 1 + \frac{S_{CN}^P}{\beta (I_{PN}^P + N_0^{CN})} \right)$$

$$+ p_0 (1 - P_f) \log \left( 1 + \frac{S_{CN}^P}{\beta N_0^{CN}} \right) \ \text{bits/s/Hz},$$  

(22)

C. Total Achievable Information Rates

The maximum information rates than can be achieved in our considered transmission scenario, is the sum-rates $R_{\text{sum}}$:

$$R_{\text{sum}} = R_{PN}^P + R_{CN}^P.$$  

(23)

Assuming $N_0^{PN} = N_0^{CN} = N_0$, $S_{PN}^P = S_{CN}^P = \bar{S}$ and defining $\gamma = \frac{\bar{S}}{N_0}$, according to (16) and (22), the sum-rates
(23) becomes:

\[
R_{\text{sum}} = p_1 (1 - P_m) \log \left(1 + \frac{\gamma}{\beta}ight) + p_0 (1 - P_f) \log \left(1 + \frac{\beta}{\gamma}ight) + p_1 P_m \log \left(1 + \left(\frac{\beta}{\beta} + \gamma^{-1}\right)^{-1}\right) + p_1 P_m \log \left(1 + \left(\frac{\beta}{\beta} + \gamma^{-1}\right)^{-1}\right) \text{ bits/s/Hz.}
\]  

(24)

In (24), the last two terms corresponds to the achievable throughputs for the primary and the cognitive network, respectively, in the fraction of DOF (i.e., region 2 in Fig. 3) where the two networks are transmitting simultaneously.

Finally, we notice that under an ideal \(^3\) (not practical) spectrum sensing characterized by \(P_m = 0\) and \(P_f = 0\), from (15) we get \(\beta = 1 - \alpha = p_0\) and the sum-rates in (24) reduces to:

\[
R_{\text{sum}} = p_1 \log \left(1 + \frac{\gamma}{\beta}ight) + p_0 \log \left(1 + \frac{\gamma}{\beta}ight) \text{ bits/s/Hz},
\]

(25)

which is nothing but the sum of rates obtained respectively in equations (11) and (17), for the primary and cognitive network with an ideal spectrum sensing.

Finally, note that by inserting a given set of parameters \((P_f, P_m)\) (characterizing a given spectrum sensing technique) in (24), one can derive the sum-rates associated to the deployed spectrum sensing technique.

V. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide numerical results to evaluate the performance provided by the proposed spectrum sensing method in comparison with conventional techniques [2], [3], [5]. We focus on the achievable information rates associated to improved and classical ED based spectrum sensing. Throughout the simulations, the transmitted power for both primary and cognitive transmitter is normalized to one and the channel bandwidth is also normalized to one. The \textit{a priori} probability on the presence of a primary user is \(p_1 = P(H^N_1) = 0.2\). For spectrum sensing, we consider the non-cooperative (i.e., with one CR) and the cooperative scheme with 12 cognitive users. However, after the sensing period, once the CR is allowed communicate with its receiver, we assume only one cognitive device is involved for data transmission. The maximum acceptable values \(P_{f_{\text{max}}}^c\) and \(P_{m_{\text{max}}}^c\) in the optimization problem (10) are respectively equal to 0.3 and 0.5 for the non-cooperative spectrum sensing and respectively equal to 0.1 and 0.3 for the cooperative spectrum sensing. The performance evaluation is performed over the BSC channel model with a transition probability of 0.005 as sensing channels and the AWGN channel model as data transmission channels for either the primary or the cognitive networks.

Fig. 4 plots the modified miss detection probability \(\tilde{P}_m\) for different spectrum sensing configurations versus the threshold value \(\zeta\) used in the ED for different spectrum sensing configurations. This enables us to find numerically the solutions of the cost function considered in (10). We have denoted by \(P_{\text{opt}}\) the optimal \(\zeta\) values minimizing the cost function (10). We have also denoted by points A and B over each curve the \(\zeta\) values corresponding to the maximal values \(P_{m_{\text{max}}}^c\) and \(P_{f_{\text{max}}}^c\) in (10). Notice that \(\zeta\) values corresponding to points A over each curve are those used in a conventional ED-based spectrum sensing.

Figure 5 illustrates the ROC curves for the three considered spectrum sensing configurations. Note that in this figure, \(P_{\text{opt}}\) indicates the optimal operating point \((P_{f_{\text{opt}}}^c, P_{m_{\text{opt}}}^c)\) corresponding to the optimal threshold \(\zeta_{\text{opt}}\) found numerically in Fig. 4.

We now analyze the total achievable information rates provided by the primary and the cognitive network where the spectrum sensing is based on either the improved or the conventional method. Fig. 6 shows the total (i.e., provided by both the primary and the cognitive network) information rates (in bits/s/Hz) versus the SNR (in dB), obtained by adopting improved and conventional spectrum sensing approaches. For comparison, we also display the upper bound on the information rates provided by an ideal spectrum sensing and also the information rate of the primary network. The first observation the large gain in the achievable information rates provided by using a cognitive network. Furthermore, the figure clearly shows the sub-optimality of conventional spectrum sensing in terms of achievable information rates compared to the rates provided by our proposed improved spectrum sensing. It can be observed that the conventional information

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1 \text{It is important to notice that the assumption } P_m = 0 \text{ and } P_f = 0 \text{ is not realistic since the point } (P_f = 0, P_m = 0) \text{ does not belong to the } \text{ROC curve of spectrum sensing. However, we have made this assumption here to get more insights about the sum-rates of (24) when spectrum sensing is ideal.}
rates are about 4.3 dB (at a sum-rate of 6 bits/s/Hz) of SNR far from the rates achieved by the ideal spectrum sensing. We note that by adopting the improved spectrum sensing, the above SNR gap is reduced by about 0.6 dB.

VI. CONCLUSION

The problem of spectrum sensing and its associated achievable throughputs in cognitive networks was investigated. Whereas conventional techniques only considers the presence of primary users in their metric formulation, we proposed new spectrum sensing metrics detecting both the absence and the presence of primary users. This framework led to an improved selection of spectrum sensing parameters. In a first step, we derived the expression of the achievable information rates of a CR, based on the improved and the conventional energy detection spectrum sensing. We also derived the total achievable throughputs for the primary and the cognitive networks. Our numerical results indicated that conventional ED is sub-optimal in terms of achievable information rates. It was shown that the rates achieved by the improved detector are very close to those provided by the ideal (interference-free) spectrum sensing. This performance improvement was obtained just by changing the cost function used for setting the threshold value in energy detection, i.e., without requiring additional complexity in the cognitive terminal.

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REFERENCES