In the name of God

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STOCHASTIC PROCESSES

Exercise Set 7

(Date Due: 1401/02/25)

1. Show that for stationary time series, $\alpha(t)$, power spectrum is given by $S(\omega) = \tilde{\alpha}(\omega)\tilde{\alpha}(\omega')^T\delta_D(\omega-\omega')$.

2. Suppose that for an isotropic stochastic field in D-dimension $\alpha \equiv \frac{f}{\sigma_0}$, $\vec{\eta} = \vec{\nabla}\alpha$ and $\xi = \nabla^2\alpha$ Show that:

$$\langle\alpha^2\rangle = 1$$

$$\langle\eta_1^2\rangle = \langle\eta_2^2\rangle = \langle\eta_3^2\rangle = \frac{1}{D}\frac{\sigma_1^2}{\sigma_0^2}$$

$$\tilde{\xi}_{ij} \equiv \xi_{ij} + \frac{1}{D}\frac{\sigma_1^2}{\sigma_0^2}$$

$$\langle\tilde{\xi}_{11}^2\rangle = \langle\tilde{\xi}_{22}^2\rangle = \langle\tilde{\xi}_{33}^2\rangle = \frac{3}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}\left(1-\frac{D+2}{3D}\gamma^2\right)$$

$$\langle\tilde{\xi}_{11}\tilde{\xi}_{22}\rangle = \langle\tilde{\xi}_{11}\tilde{\xi}_{33}\rangle = \langle\tilde{\xi}_{22}\tilde{\xi}_{33}\rangle = \frac{1}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}\left(1-\frac{D+2}{D}\gamma^2\right)$$

$$\langle\tilde{\xi}_{12}^2\rangle = \langle\tilde{\xi}_{13}^2\rangle = \langle\tilde{\xi}_{23}^2\rangle = \frac{1}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}$$

3. Compute $\langle|\eta_i|\rangle$ for 1 and 2 and 3 dimensions.

4. For a random-walk suppose that probability distribution of each jump is represented by $p(s) = \frac{1}{1+s^\alpha}$, in this case:

(a) Determine the $p(x)$ after $N-$step.
(b) Compute $\langle x\rangle_N$
(c) Compute $\langle x^2\rangle_N - \langle x\rangle_N^2$
(d) What about $p(x)$ for $N\to\infty$?

5. For standard random-walk model we find $\sigma_x(t) \sim t^\alpha$ with $\alpha = 0.5$. Explain how one can derive dispersion for random-walk position with $\alpha \neq 0.5$.

Good luck, Movahed