In the name of God

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STOCHASTIC PROCESSES

Exercise Set 6

(Due Date: 1401/02/10)

1. Bias definition: The relation between Unweighted TPCF and weighted TPCF can be considered as:

$$\Psi_{fg}(R) = \mathcal{B}_{fg} C(R)$$

(a) For Up-crossing feature determine $\mathcal{B}$ for 1+1-D stochastic field.

(b) For a sharp clipping feature, we have $f \equiv A\Theta(\alpha - \vartheta)$. Here A is a normalization coefficient. The average value of sharp clipping in a Gaussian random field can be written by:

$$\langle f(\alpha)\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A\Theta(\alpha - \vartheta) e^{-\alpha^2/2} d\alpha$$

Now, we expand $\Theta(\alpha - \vartheta)$ in terms of Hermite polynomials defined by $H_n = e^{x^2/2}(d/dx)^n e^{-x^2/2}$ with $\langle H_n(x) H_m(x)\rangle \delta_{nm} m!$. Therefore we have

$$A\Theta(\alpha) = \sum_{k=0}^{\infty} \frac{J_k}{k!} H_k(\alpha)$$

Show that

$$J_k = \frac{\vartheta H_{k-1}(\vartheta)}{\vartheta \sqrt{\pi/2}\exp(\vartheta^2/2)\mathrm{erfc}(\vartheta/\sqrt{2})}$$

(c) Show $\lim_{\vartheta\to\infty} J_k = \vartheta^k$

(d) Show $\langle \Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1 + \Psi(R) = \sum_{k=0}^{\infty} \frac{J_k^2}{k!} C_{\alpha\alpha}^k(R)$, where $R = |r - r'|$.

(e) For $R \to \infty$ show $\langle \Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1 + \Psi(R) \sim J_1^2 C_{\alpha\alpha}(R)$. Here $J_1 = \vartheta^2$.

Good luck, Movahed
