In the name of God

Department of Physics
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ADVANCED STATISTICAL MECHANICS I

Exercise Set 9

(Due Date: 1400/11/10)

1. Determine the following thermodynamical potentials of Ultra-relativistic Bose Gas: F, H, G. Also Compute C_V and C_P for mentioned system.

2. For an ideal Bose gas, we obtained that at $T = T_c$, the fugacity is equal to one ($z = 1$), accordingly, we can determine the value of T_c (see the lecture note and notice to $N = \int_0^\infty d\epsilon g(\epsilon) n_{BE}(\epsilon)$, where $n_{BE}(\epsilon)$ is called BE distribution given by $n_{BE}(\epsilon) = \frac{1}{\exp(\beta(\epsilon-\mu))-1}$, for $T = T_c$ we have

$$N = \int_0^\infty d\epsilon g(\epsilon) \frac{1}{\exp(\frac{\epsilon}{k_B T_c} - 1)}$$

Now consider, our Bose system contains two level of energy, the particles in ground state have $\epsilon_0 = p^2/(2m)$ and those particles in excited state have $\epsilon = \epsilon_0 + \Delta$, for this case, compute T_c (Hint: at first determine the density of sate for the ground state $g_0(\epsilon)$ and for excited state $g_{\text{excited}}(\epsilon)$, then set $z = 1$).

3. According to the statistical definition of pressure, determine the equation of state parameter of ideal photon gas.

4. For ideal fermi gas, show that

$$\frac{PV}{Nk_BT} = \sum_{\ell=1}^{\infty} (-1)^{\ell-1} a_\ell \left(\frac{\lambda^3}{g_s V/N}\right)^{\ell-1}$$

and

$$C_V = \frac{3}{2} N k_B \sum_{\ell=1}^{\infty} (-1)^{\ell-1} \frac{5-3\ell}{2} a_\ell \left(\frac{\lambda^3}{g_s V/N}\right)^{\ell-1}$$

and compute a_ℓ.

5. Derive equations 8.1.37 and 8.1.38

Good luck, Movahed