In the name of God

Department of Physics Shahid Beheshti University

ADVANCED TOPICS IN STATISTICAL PHYSICS II

Exercise Set 6

(Date Due: 1395/04/10)

1. According to following definition:

$$\int_{-\infty}^{+\infty} x^n \exp[-(x-\beta)^2] dx = (2i)^{-n} \sqrt{\pi} H_n(i\beta)$$

where H_n are the Hermite polynomials show that:

$$M_n(x',t,\tau) = \left[-i\sqrt{D^{(2)}(x',t)\tau}\right]^n H_n\left\{\frac{1}{2}iD^{(1)}(x',t)\sqrt{\tau/D^{(2)}(x',t)}\right\}$$

also show that above equation causes to correct function for $D^{(n)}$.

2. Calculate the Moments: Using Green's function approach show:

 \mathbf{A} :

$$M_i(t) = \langle x_i(t) \rangle = G_{ij} x_j$$

 \mathbf{B} :

$$\sigma_{ij} = \langle [x_i(t) - \langle x_i \rangle] [x_j(t) - \langle x_j \rangle] \rangle = \int_0^t G_{ik}(t') G_{js}(t') g_{ks}(t') g_{ks$$

 ${\bf C}:$

$$\dot{\sigma}_{ij} = -\xi_{ik}\sigma_{kj} - \xi_{jk}\sigma_{ki} + g_{ij}$$

D :

$$\ddot{\sigma}_{ij} = -\xi_{il}G_{lk}G_{js}g_{ks} - G_{ik}\xi_{jl}G_{ls}g_{ls}$$

3. Using the value of $D^{(1)}, D^{(2)}, D^{(3)}, D^{(4)}$, compute $\langle x^4 \rangle$ as a function of $\langle x^3 \rangle$ and $\langle x^2 \rangle$ for data that you have.

4. By computing the $D^{(1)}$ and $D^{(2)}$ for $\Delta x \equiv x(t + \tau) - x(t)$, compute the correlation function, $C_x(\tau) = \langle x(t + \tau)x(t) \rangle$. Compare your results with that of given directly by data.

Good luck, Movahed