In the name of God

## Department of Physics Shahid Beheshti University

## ADVANCED TOPICS IN STATISTICAL PHYSICS II

## Exercise Set 6

(Date Due: 1395/04/10)

1. According to following definition:

$$
\int_{-\infty}^{+\infty} x^{n} \exp \left[-(x-\beta)^{2}\right] d x=(2 i)^{-n} \sqrt{\pi} H_{n}(i \beta)
$$

where $H_{n}$ are the Hermite polynomials show that:

$$
M_{n}\left(x^{\prime}, t, \tau\right)=\left[-i \sqrt{D^{(2)}\left(x^{\prime}, t\right) \tau}\right]^{n} H_{n}\left\{\frac{1}{2} i D^{(1)}\left(x^{\prime}, t\right) \sqrt{\tau / D^{(2)}\left(x^{\prime}, t\right)}\right\}
$$

also show that above equation causes to correct function for $D^{(n)}$.
2. Calculate the Moments: Using Green's function approach show:

A :

$$
M_{i}(t)=\left\langle x_{i}(t)\right\rangle=G_{i j} x_{j}
$$

B :

$$
\sigma_{i j}=\left\langle\left[x_{i}(t)-\left\langle x_{i}\right\rangle\right]\left[x_{j}(t)-\left\langle x_{j}\right\rangle\right]\right\rangle=\int_{0}^{t} G_{i k}\left(t^{\prime}\right) G_{j s}\left(t^{\prime}\right) g_{k s}
$$

C :

$$
\dot{\sigma}_{i j}=-\xi_{i k} \sigma_{k j}-\xi_{j k} \sigma k i+g_{i j}
$$

D :

$$
\ddot{\sigma}_{i j}=-\xi_{i l} G_{l k} G_{j s} g_{k s}-G_{i k} \xi_{j l} G_{l s} g_{l s}
$$

3. Using the value of $D^{(1)}, D^{(2)}, D^{(3)}, D^{(4)}$, compute $\left\langle x^{4}\right\rangle$ as a function of $\left\langle x^{3}\right\rangle$ and $\left\langle x^{2}\right\rangle$ for data that you have.
4. By computing the $D^{(1)}$ and $D^{(2)}$ for $\Delta x \equiv x(t+\tau)-x(t)$, compute the correlation function, $C_{x}(\tau)=$ $\langle x(t+\tau) x(t)\rangle$. Compare your results with that of given directly by data.

Good luck, Movahed

