In the name of God

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ADVANCED TOPICS IN STATISTICAL PHYSICS II

Exercise Set 6

(Date Due: 1393/01/30)

1. The same as my calculation regarding $N_3(\nu)$, $N_2(\nu)$ and $N_1(\nu)$ for $1+3$D Gaussian stochastic field, calculate the relevant quantities for $1+2$D and $1+1$D in the Gaussian case.

2. Suppose that for a $1+3$D Gaussian random field, power spectrum is given by:

$$P(k) = Ak^n \exp\left[-\frac{1}{2}\left(k^2\lambda^2 - \frac{1}{k^2\mu^2}\right)\right]$$

here λ and μ are short-wavelenght and long-wavelenght cutoff respectively. Compute $N_1(\nu)$, $N_2(\nu)$ and $N_3(\nu)$.

3. For a stochastic field in D-dimension, show that (see ref. T. Matsubara, The Astrophysical Journal, 584:1-33, (2003)):

$$N_1(\nu) = \frac{1}{\pi D}\frac{\sigma_1}{\sigma_0}e^{-\nu^2 2}\left\{1 + \left[\frac{S^{(0)}}{6}H_3(\nu) + \frac{S^{(1)}}{3}H_1(\nu)\right]\sigma_0 + \mathcal{O}(\sigma_0^2)\right\}$$

where $S^{(0)} = \frac{\langle f^3\rangle}{\sigma_0^4}$, $S^{(1)} = -\frac{3}{4}\frac{\langle f^2\nabla^2 f\rangle}{\sigma_0^2\sigma_1^2}$. Use $\mathcal{R}(k,l) \equiv \langle\left(\frac{\partial}{\partial\alpha}\right)^k\left(\frac{\partial}{\partial\eta_1}\right)^l\delta(\alpha-\nu)\eta_1|\rangle = \frac{h_{l-2}}{\pi}\left(\frac{\sigma_1}{\sqrt{D}\sigma_0}\right)^{1-l}e^{-\nu^2/2}H_k(\nu)$

Good luck, Movahed