In the name of God

Department of Physics
Shahid Beheshti University

ADVANCED TOPICS IN STATISTICAL PHYSICS II

Exercise Set 5

(Date Due: 1394/03/01)

1. Computational program: Here we are going to compute Kramers-Moyal coefficients for simulated data given in previous set of problem.

A : Compute $D^{(1)}(x,t)$ and $D^{(2)}(x,t)$ and plot them as a function of x. (Hint: at first you should determine the markov length scale and set $t = t_{Markov}$.)

B : Show that $D^{(4)}(x,t)$ is very small in comparison with $D^{(2)}(x,t)$.

2. According to forward solution, and suppose that $D^{(4)}(x,t) = 0$, show that:

$$p(x,t+\tau|x',t) = \left[1 - \frac{\partial}{\partial x}D^{(1)}(x,t)\tau + \frac{\partial^2}{\partial x^2}D^{(2)}(x,t)\tau\right]\delta(x-x')$$

has the following solutions:

A :

$$p(x,t+\tau|x',t) = \frac{1}{2\sqrt{\pi D^{(2)}(x',t)\tau}}\exp\left(-\frac{[x-x'-D^{(1)}(x',t)\tau]^2}{4D^{(2)}(x',t)\tau}\right)$$

B :

$$p(x,t+\tau|x',t) = \frac{1}{2\sqrt{\pi D^{(2)}(x,t)\tau}}\exp\left(-\frac{\partial}{\partial x}D^{(1)}(x,t)\tau + \frac{\partial^2}{\partial x^2}D^{(2)}(x,t)\tau - \frac{[x-x'-(D^{(1)}(x,t)-2\frac{\partial}{\partial x}D^{(2)}(x,t))\tau]^2}{4D^{(2)}(x,t)\tau}\right)$$

Good luck, Movahed