Introduction to Cosmological Codes: CAMB/ CosmoMC

Hossein Mos'hafi

SBU,
Spring 2018

Outline

- Introduction to CAMB
- Introduction to CosmoMC

CAMB

- **C**ode for **A**nisotropies in the **M**icrowave **B**ackground
- Main authors: Antony Lewis, Anthony Challinor
- Language: Fortran + Python wrapper
- Includes: Halofit; scalar, vector and tensor modes; polarization; lensed CMB and lensing potential
- Internal parallelization of loops; support for adiabatic or isocurvature initial conditions; estimates bispectrum; controllable accuracy
- It solves Boltzmann equations and computes: **CMB power spectra** ,**Transfer functions**, **Matter power spectrum**
- CAMB is available at **camb.info** or on GitHub: **github.com/cmbant/CAMB.git**
- Also there is web interface for CAMB on the LAMBDA website:

  https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

Einstein-Boltzmann Eqs.

Liouville term ⟷ Collision term

Einstein+Boltzmann equ.

$$1)\ \dot{\Theta}_T + ik\mu\Theta_T + \dot{\Phi} + ik\mu\Psi = -\dot{\tau}[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi]$$

$$2)\ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}[-\Theta_P + \frac{1}{2}(1-\mathcal{P}_2(\mu))\Pi]$$

$$\Pi = \Theta_{T2} + \Theta_{P2} + \Theta_{P0}$$

$$3)\ \dot{\mathcal{N}} + ik\mu\mathcal{N} + \dot{\Phi} + ik\mu\Psi = 0$$

Photon/Neutrinos

EB equs. with fluid approximation (Euiler+continuity)

$$4)\ \dot{\delta}_c = -ikv_c - 3\dot{\Phi}$$

$$5)\ \dot{\delta}_b = -ikv_b - 3\dot{\Phi}$$

$$6)\ \dot{v}_c = -Hv_c - ik\Psi$$

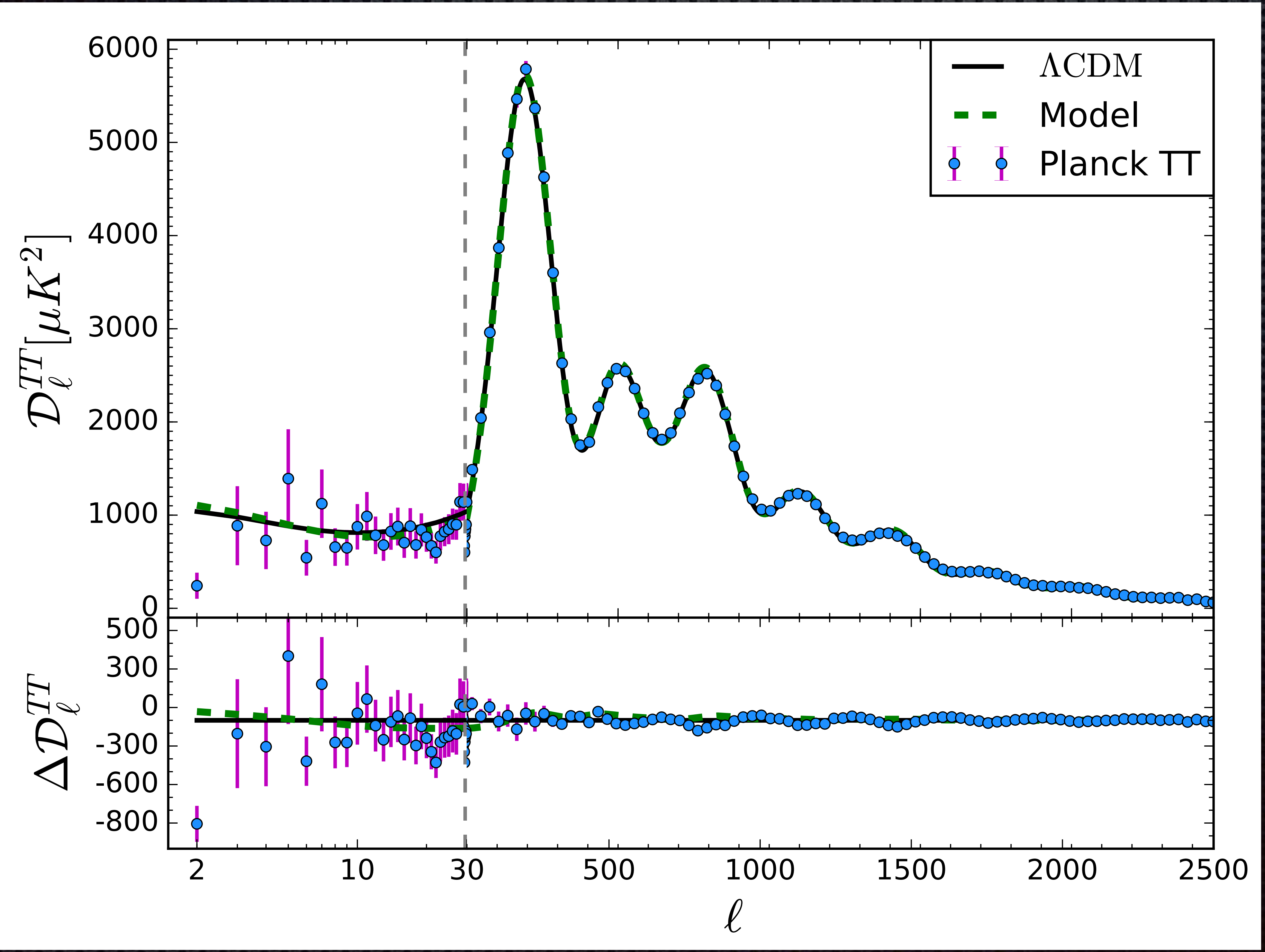
$$7)\ \dot{v}_b = -Hv_b - ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1], \quad \frac{1}{R} \equiv \frac{4\rho_\gamma^{(0)}}{3\rho_b^{(0)}}$$

Baryon/CDM

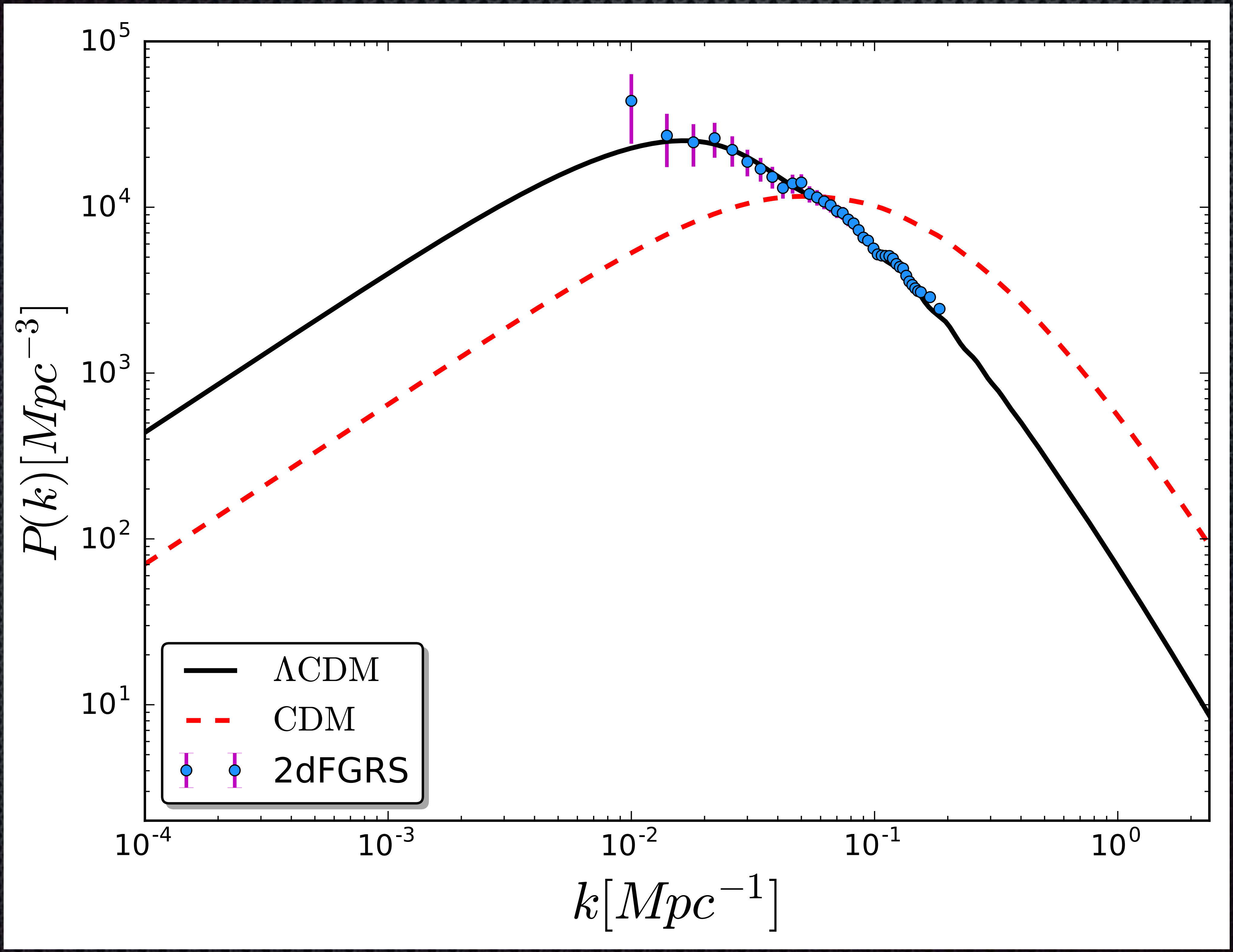
CAMB Outputs

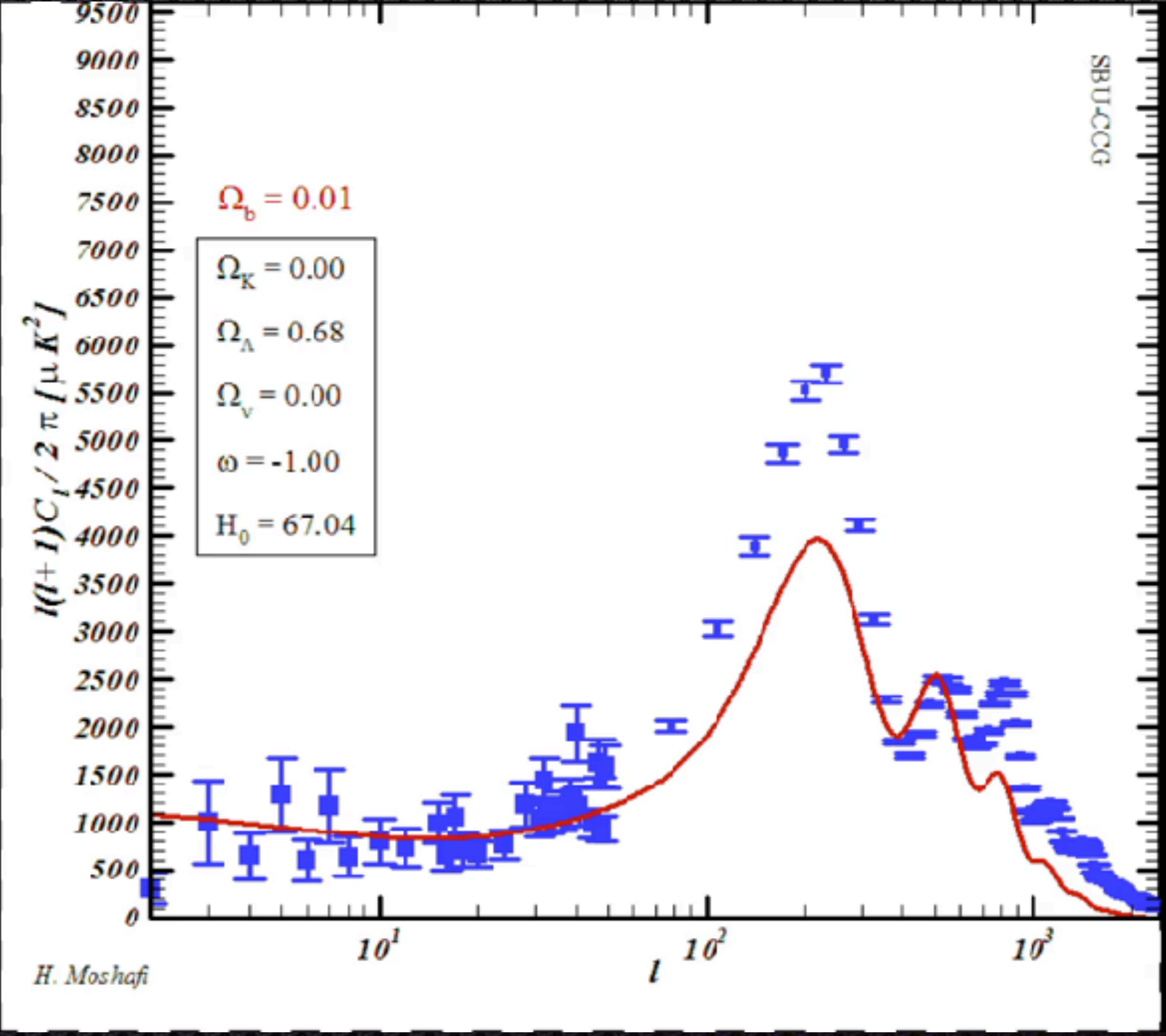
- Outputs:
  - Spectrum of the cosmic microwave background anisotropies:
    - root_scalCls.dat: {l, CTT, CEE, CTE, [Cphiphi, CphiT]}
    - root_lensedCls.dat: {l, CTT, CEE, CBB, CTE}
    - root_lenspotentialCls.dat: {l, CTT, CEE, CBB, CTE, Cdd, CdT, CdE}
    - root_tensCls.dat: {l, CTT, CEE, CBB, CTE}
- Matter power spectrum : root_matterpower.dat : {k, Pk}
- Transfer function for all particle perturbations: root_transfer_out.dat: {k/h, Delta_CDM/k^2, Delta_b/k^2, Delta_r/k^2, Delta_nu/k^2, Delta_tot/k^2}

Temperature anisotropies power spectrum

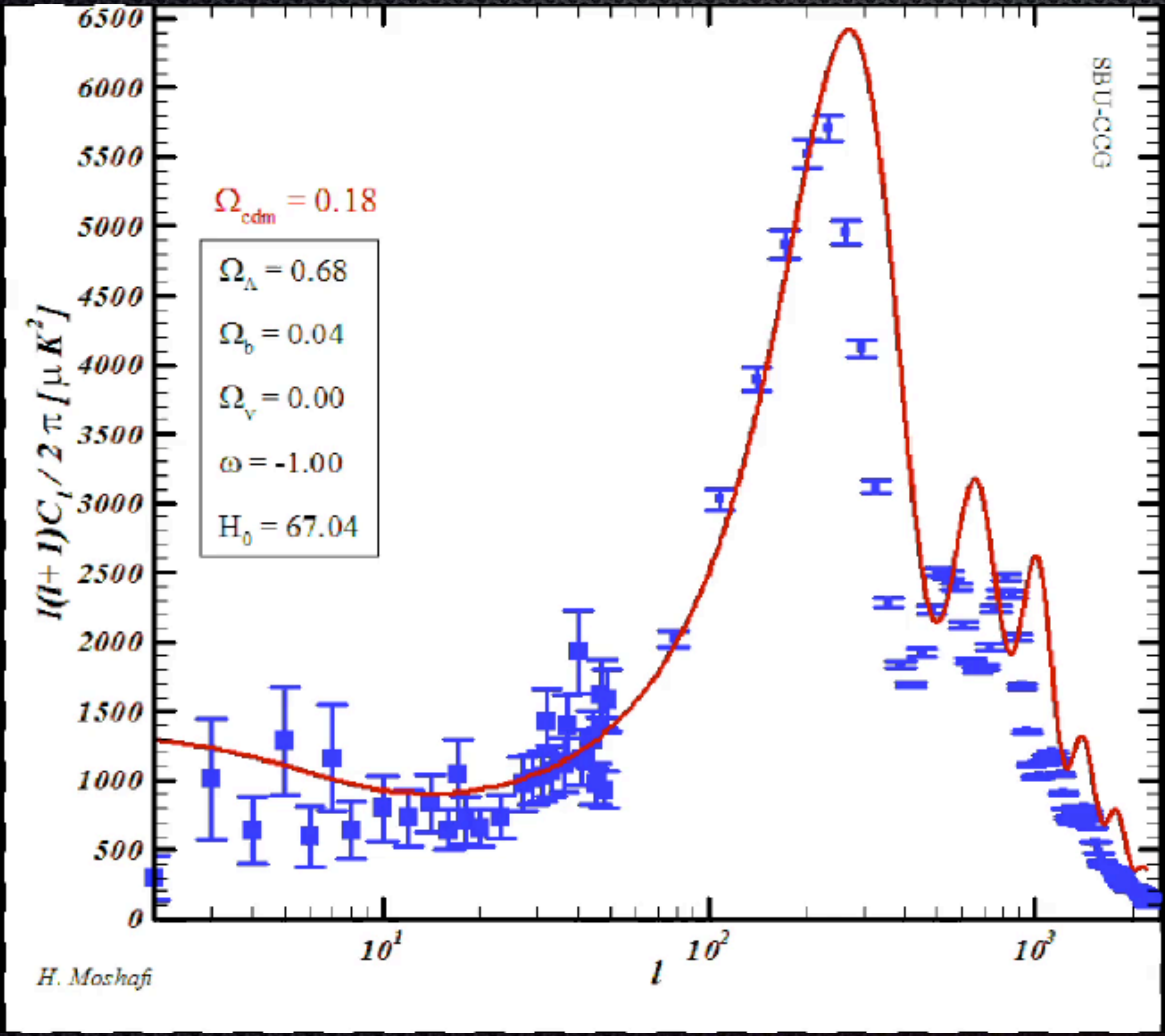


Matter power spectrum

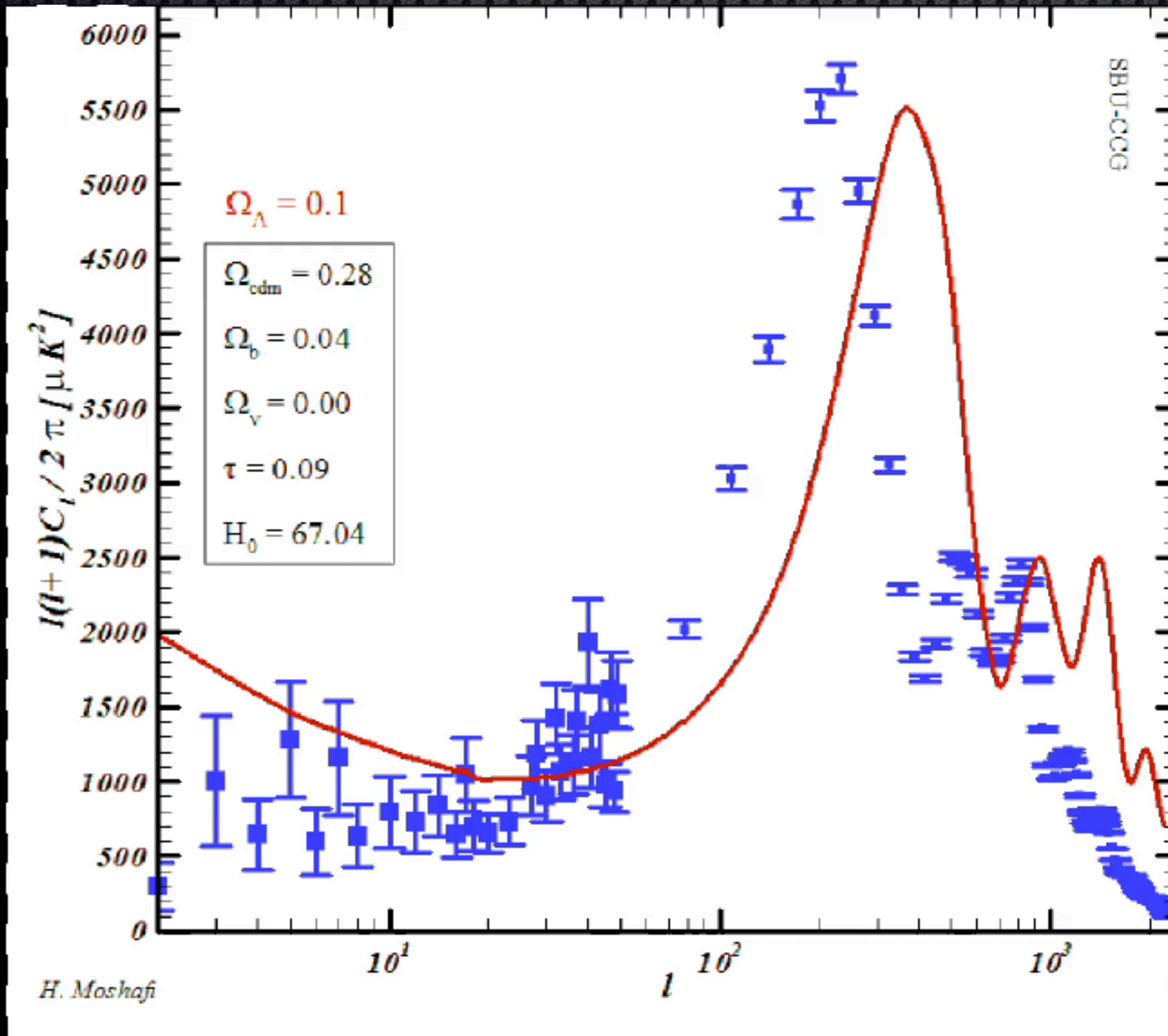




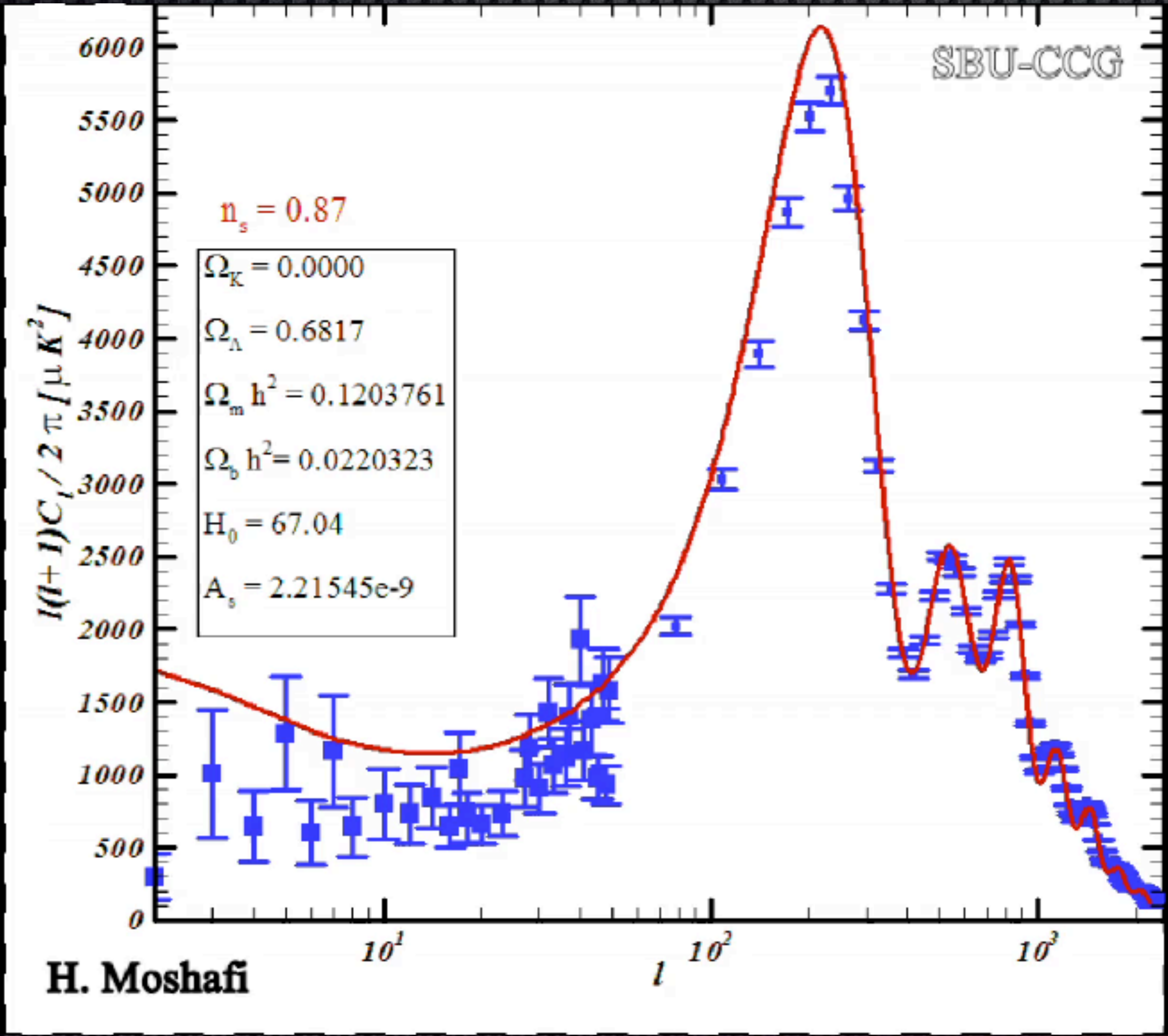
9500
9000
8500
8000
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
$l(l+1)C_l / 2\pi$ [μK^2]
Ω_b = 0.01
Ω_K = 0.00
Ω_Λ = 0.68
Ω_ν = 0.00
ω = -1.00
H_0 = 67.04
SBU-CCG
10^1
10^2
10^3
l
H. Moshafi



6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
$l(l+1)C_l / 2\pi\ [\mu K^2]$
$\Omega_{cdm} = 0.18$
$\Omega_\Lambda = 0.68$
$\Omega_b = 0.04$
$\Omega_\nu = 0.00$
$\omega = -1.00$
$H_0 = 67.04$
SBU-CCG
10^1
10^2
10^3
l
H. Moshafi



6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
$l(l+1)C_l / 2\pi\ [\mu K^2]$
$\Omega_\Lambda = 0.1$
$\Omega_{cdm} = 0.28$
$\Omega_b = 0.04$
$\Omega_\nu = 0.00$
$\tau = 0.09$
$H_0 = 67.04$
10^1
10^2
10^3
l
SBU-CCG
H. Moshafi


SBU-CCG
$n_s = 0.87$
$\Omega_K = 0.0000$
$\Omega_\Lambda = 0.6817$
$\Omega_m h^2 = 0.1203761$
$\Omega_b h^2 = 0.0220323$
$H_0 = 67.04$
$A_s = 2.21545e-9$
$l(l+1)C_l / 2\pi\ [\mu K^2]$
l
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
10^1
10^2
10^3
H. Moshafi

CAMB program files

- bessels.f90
- camb.f90
- cmbmain.f90
- constants.f90
- cosmorec.f90
- equations.f90
- halofit.f90
- hyrec.f90
- inidriver.f90
- inifile.f90
- lensing.f90
- Matrix_utils.f90
- modules.f90
- power_tilt.f90
- recfast.f90
- reionization.f90
- SeparableBispectrum.F90
- sigma8.f90
- subroutines.f90
- tester.f90
- utils.f90
- writefits.f90

CAMB program files

drivers: can be used to run different tests

- bessels.f90
- camb.f90
- cmbmain.f90
- constants.f90
- cosmorec.f90
- equations.f90
- halofit.f90
- hyrec.f90
- inidriver.f90
- inifile.f90
- lensing.f90
- Matrix_utils.f90
- modules.f90
- power_tilt.f90
- recfast.f90
- reionization.f90
- SeparableBispectrum.F90
- sigma8.f90
- subroutines.f90
- tester.f90
- utils.f90
- writefits.f90

CAMB files:

drivers: can be used to run different tests

sigma8.f90 | inidriver.f90 | tester.f90

- **Inidriver**: Reads in parameters from a file of name/value pairs and calls CAMB. Modify this file to generate grids of models, change the parametrization, etc.
- **sigma8, tester**: Sample programs. These are supplied showing how to use CAMB from your own programs.

CAMB program files

without cosmological logic

- bessels.f90
- camb.f90
- cmbmain.f90
- constants.f90
- cosmorec.f90
- equations.f90
- halofit.f90
- hyrec.f90
- inidriver.f90
- inifile.f90
- lensing.f90
- Matrix_utils.f90
- modules.f90
- power_tilt.f90
- recfast.f90
- reionization.f90
- SeparableBispectrum.F90
- sigma8.f90
- subroutines.f90
- tester.f90
- utils.f90
- writefits.f90

CAMB program files

without cosmological logic

- 6 programs with useful routines
- Never change them !

subroutines.f90

Matrix_utils.f90

bessels.f90

inifile.f90

writefits.f90

utils.f90

CAMB program files

with cosmology inside

bessels.f90	camb.f90	cmbmain.f90	constants.f90
cosmorec.f90	equations.f90	halofit.f90	hyrec.f90
inidriver.f90	inifile.f90	lensing.f90	Matrix_utils.f90
modules.f90	power_tilt.f90	recfast.f90	reionization.f90
SeparableBispectrum .F90	sigma8.f90	subroutines.f90	tester.f90
utils.f90	writefits.f90		

CAMB program files

with cosmology inside

- **cmbmain.f90**: The main subroutine that does integrations, etc. Encompasses CMBFAST's cmbflat and cmbopen.
- **camb.f90**: Main wrapper routines for running CAMB in your programs.
  - Add "**use camb**" to your programs and **call CAMB_GetResults** to generate output from a set of model parameters.
  - use **CAMB_GetAge** to compare the age of a model, and **CAMB_GetCls** to retrieve the computed Cls.
- **recfast.f90**: RECFAST integrator for cosmic recombination of hydrogen and helium. It could be replaced by more detailed codes : **CosmoRec** and **HyRec**.

CAMB program files

with cosmology inside

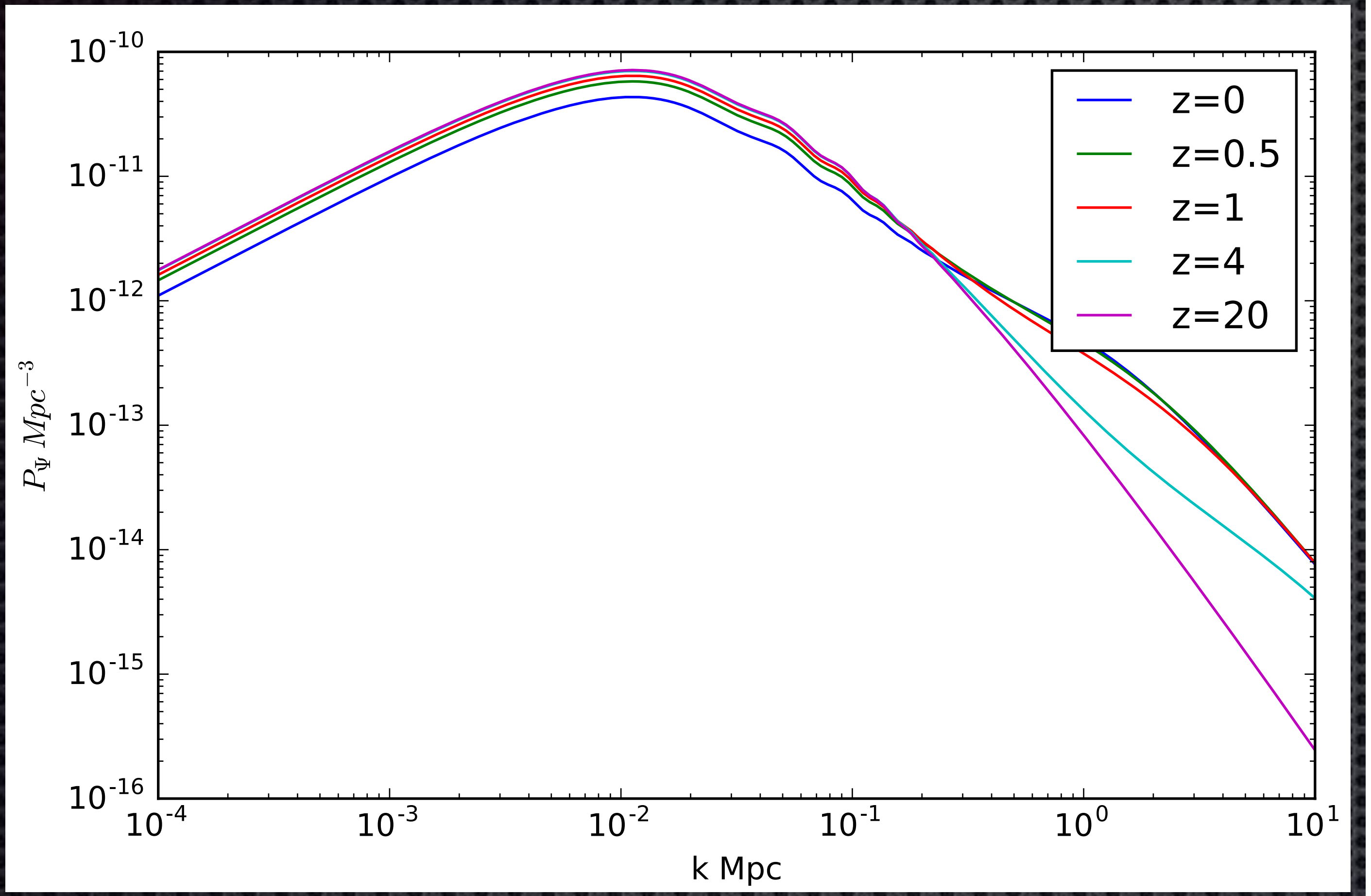
- **power_tilt.f90**: This file defines a module called InitialPower that returns the initial power spectra. Change this file to use your own initial power spectrum.
- **reionization.f90**: This file defines a module called Reionization that parametrizes the deionization history and supplies a function Reionization_xe that gives xe as a function of redshift.
- **halofit.f90**: Implements the NonLinear modules, to calculate non linear scalings of the matter power spectrum as a function of redshift. This models can be replaced to use different non linear fitting methods.

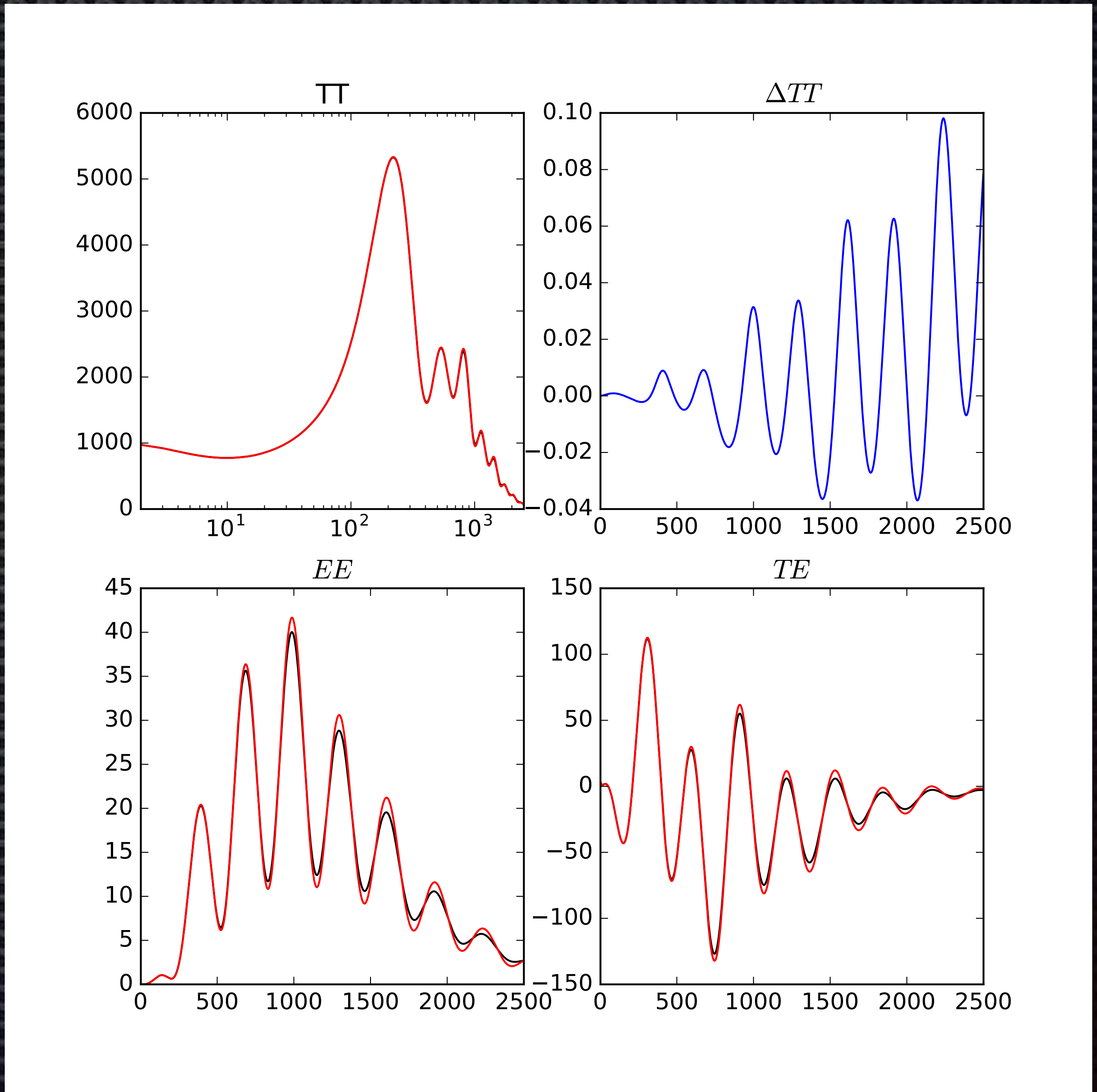
CAMB program files

with cosmology inside

- **lensing.f90**: Lensing module for computing the lensed CMB power spectra from unlicensed spectra and a lensing power spectrum.
- **SeparableBispectrum.f90**: Implements calculation of simple separable primordial bispectra, specifically the local constant f_NL model, and the CMB lensing bispectrum due to the linear temperature and polarization cross-correlation with the lensing potential.
- **equations.f90**: File containing background and perturbation evolution equations. The perturbations equations used are derived in the covariant approach, fixing to the CDM (zero acceleration) frame, which are equivalent to the synchronous gauge equations.

CAMB: Python wrapper





Modification Examples

- Initial power $$\mathcal{P}_{\mathcal{R}} = A_s \left(\frac{k}{k_p} \right)^{n_s - 1} \times \left(1 + \alpha \cos\left(\frac{k}{k_p}\right) \right)$$

- Background $$w(a) = w_0 + w_1 (1 - a)$$

Standard Model

- Standard model of cosmology is current simplest framework to describe the cosmological observations.
- The standard cosmological model is **ΛCDM model**. In this model, the Universe contains dark matter (and baryonic) , and the accelerated expansion rate is due to a cosmological constant (**Λ**).
- Also the gravity is defined by FLRW metric.

Standard Model

- There are “6” parameters in the ‘base’ ΛCDM model:

$$\left\{\Omega_b h^2, \Omega_c h^2, H_0, \tau_{\mathrm{re}}, A_s, n_s\right\}$$

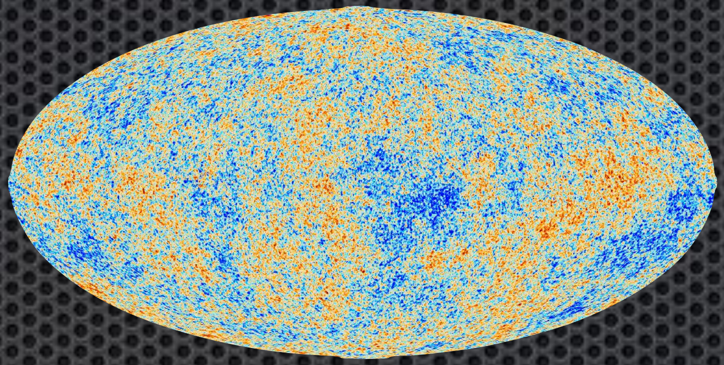
- Physical density of baryonic matter $\Omega_b h^2$
- Physical density of cold dark matter $\Omega_c h^2$
- The local expansion rate H_0
- The optical depth to reionization τ_{re}
- Amplitude of primordial scalar power spectrum A_{S}
- Spectral index of scalar power spectrum n_{S}

Beyond the Standard Model

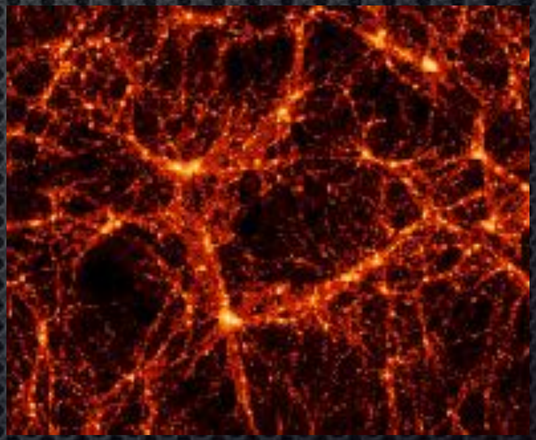
- **There are some additional parameters not included in base model.**
- The curvature of the Universe Ω_K
- The amount of matter in the form of massive neutrinos $\sum m_\nu$
- The effective number of relativistic species at recombination N_{eff}
- The equation of state of dark energy (and its time dependence) w_0, w_a
- The tensor to scalar power spectrum ratio r
- Number of e-folds N
- Non-Gaussianity f_{NL}
- Running of spectral index n_{run}
- Modified gravity parameters a

Cosmological Observations

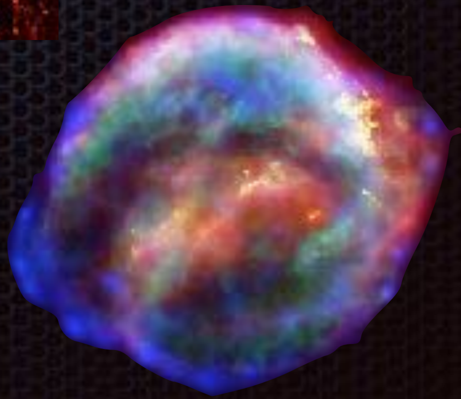
- Cosmic Microwave Background (temperature and polarization)



- Matter power spectrum (clusters, weak lensing, lyman alpha)



- Standard markers (candles, rulers, clocks,…)



Cosmological Datasets

- **CMB**. Angular power spectrum of temperature and polarization. Latest measurements by Planck satellite.
- **BAO**. Cosmic distance scale measurements using Baryon Acoustic Oscillations. Latest is BOSS.
- **SN**. Type-Ia Supernovae. Distance modulus from the luminosity distance derived by the explosion of SNIa. Latest compilation by Joint Light Curve analysis (JLA).

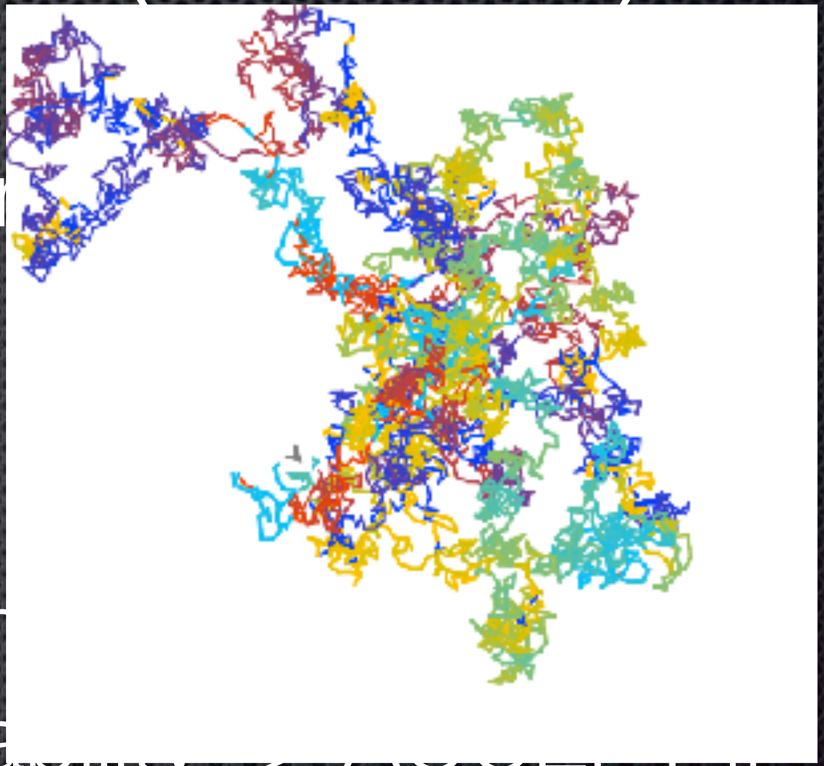
Cosmological Datasets

- **P(k)**. Measured by clustering of the galaxy distribution or distribution of Lyman- Alpha Forest.
- **H0**. Direct measurements of the expansion rate in the local Universe using cepheids, SN host galaxies, and water maser distances.
- **RSD**. Redshift space distortions. Measuring the clustering of cosmic tracers such as galaxies and Ly-alpha forest in redshift space it is possible to derive structure growth and test GR. Every galaxy survey has provided a measurement.
- Others: Weak-Lensing, SZ, …

CosmoMC

- **Cosmo**logical **M**onte **C**arlo is a parameter *sampling* code, implements the Metropolis- Hastings algorithm to sample a given parameter space.
- It is bundled up with likelihood codes from the most recent data such as CMB likelihood from Planck, BAO likelihood from BOSS or SN likelihood from JLA.
- It is written in Fortran 90.
- GetDist package analyzes Markov chains and produces parameter tables and plots.
- Source code is available at: **http://cosmologist.info/cosmomc/**

Markov Chain Monte Carlo (MCMC)



- It is a **random walk** to effectively sample a
parameter space (when the number of dime
sampling a grid is expensive)

- We use the **Metropolis-Hastings** algorithm
if a step puts you in a place with more proba
not -> ACCEPT with probability=likelihood ratio=P_{new}/P_{old})
otherwise, REJECT (try a different move from previous step)

- CosmoMC parallels MCMC processes. Independent, but communicating, MCMC going by different paths to reach the same best fit region with velocity and precision.

χ^2 and Likelihood

- The measurements in cosmological datasets are translated to **likelihoods**. The total likelihood, assuming the measurements of the experiments are not correlated, is the product of individual likelihoods.

- However in practice, CosmoMC uses the log of the likelihood (with opposite sign), which is closely related to the value of the χ^2 distribution. These are related by the equation:

$$\mathcal{L} \propto e^{-\frac{\chi^2}{2}}$$

- Where the χ^2 for simple cases is just (measurement-theory)^2/error^2, or more generally $\chi^2 = (\mathbf{D}-\mathbf{T})^t\, \mathbf{C}^{-1}\, (\mathbf{D}-\mathbf{T})$ where **D** is a vector of data measurements and **T** is the theory vector generated at each MCMC step. $\mathbf{C}$ is the **covariance matrix** whose elements are the covariances between parameters $\mathbf{C}=\{\sigma_{\theta i,\theta j}\}$, so that the diagonal elements are $\sigma^2_{\theta i}$ and the off-diagonal are $\sigma_{\theta i \theta j} = \rho_{ij}\, \sigma_{\theta i}\, \sigma_{\theta j}$

MCMC chains

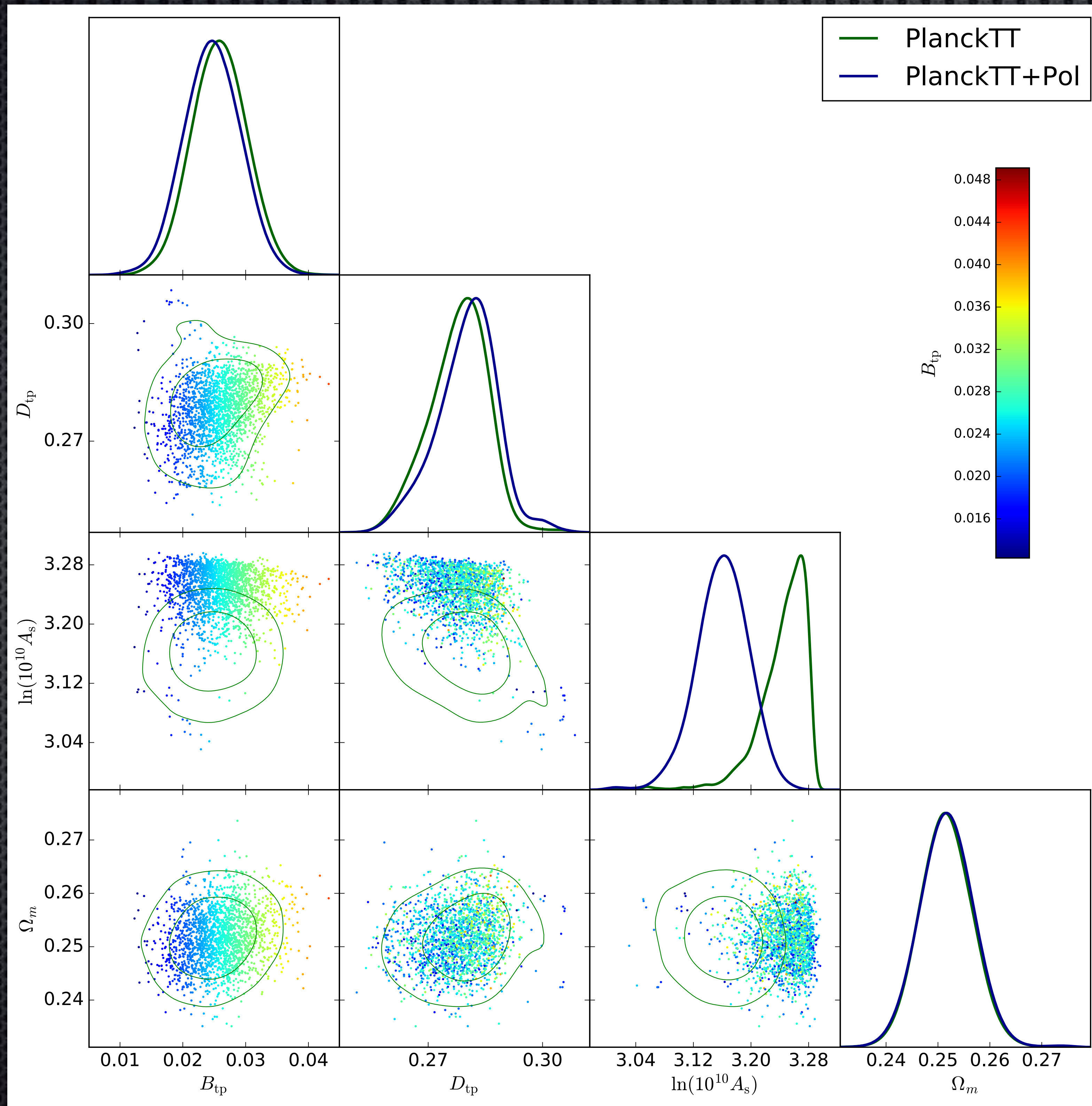
Model parameters

number of steps | -log(L)

MCMC step

0.1000000E+01	0.4759569E+03	0.2213998E-01	0.1172434E-00	0.1041655E+01	0.8053226E-01	0.3099537E+01	0.9640315E+00	0.6839050E-02	0.7806185E+00	0.2993815E+00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.1254866E+04	0.5805181E+04	0.2553392E+04	0.8182748E-03	0.2300445E+03	0.9640315E+00	0.2452879E-00	0.2456140E+02	0.2634928E+01	0.1378663E+02	0.1089967E+04
0.1453257E+03	0.1241870E+01	0.1394855E+02	0.1059208E-24	0.1480835E+03	0.1396480E+00	0.1612821E-00	0.3330813E+04	0.1016605E-01	0.8262200E+00	0.4563183E+00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
0.2000000E+01	0.4648023E+03	0.2222156E-01	0.1175487E+00	0.1041608E-01	0.8158817E-01	0.3098752E+01	0.9542759E-00	0.6831360E+02	0.6991579E+00	0.3008421E+00
0.1403954E+00	0.6451439E-03	0.9590915E-01	0.8259503E-20	0.4530261E+00	0.6117001E+00	0.9993103E-00	0.2484429E+01	0.1027374E+02	0.2217027E+01	0.1883235E+01
0.1250557E+04	0.5784877E+04	0.2545391E+04	0.8162917E-03	0.2296715E+03	0.9642759E+00	0.2453178E-00	0.2466441E+02	0.2623109E+01	0.1378339E+02	0.1089916E+04
0.1451980E+03	0.1241810E+01	0.1393709E+02	0.1059351E-24	0.1479332E+03	0.1398533E+00	0.1611777E-00	0.3339583E+04	0.1019281E-01	0.8247286E+00	0.4554918E+00
0.3541956E-01	0.1846745E+03	0.1783333E+04	0.3075932E-21	0.3528764E+00	0.3723092E+00	0.9137652E-03	0.1558944E+02	0.2500000E+00	0.9293547E+03	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.1399726E+00	0.6451439E-03	0.9556132E-01	0.8254475E-20	0.4518742E+00	0.6107360E+00	0.9984878E-00	0.2476061E+01	0.1048431E+02	0.2215397E+01	0.1873584E+01
0.1239266E+04	0.5744383E+04	0.2535189E+04	0.8134976E-03	[illegible]	[illegible]	[illegible]	0.2466045E+02	0.2638579E+01	0.1379240E+02	0.1089989E+04
0.1453501E+03	0.1241736E+01	0.1395267E+02	0.1059170E-24	[illegible]	[illegible]	[illegible]	0.3329476E+04	0.1016197E-01	0.8263077E+00	0.4563754E+00
0.3543580E-01	0.1844453E+03	0.1784278E+04	0.3073729E-21	[illegible]	[illegible]	[illegible]	0.1456813E+02	0.2500000E+00	0.9243310E+03	
0.2000000E+01	0.4623016E+03	0.2211407E-01	0.1172279E+00	[illegible]	[illegible]	[illegible]	0.9561860E-00	0.6833191E+02	0.7001935E+00	0.2998065E+00
0.1399872E+00	0.6151439E-03	0.9565592E-01	0.8254732E-20	[illegible]	[illegible]	[illegible]	0.2477396E+01	0.1044159E+02	0.2215686E+01	0.1875808E+01
0.1241554E+04	0.5752319E+04	0.2537875E+04	0.8140731E-03	0.2289583E+03	0.9661860E+00	0.2452750E-00	0.2466011E+02	0.2639893E+01	0.1379298E+02	0.1089999E+04
0.1453497E+03	0.1241744E+01	0.1395253E+02	0.1059132E-24	0.1481183E+03	0.1395938E+00	0.1612987E-00	0.3329824E+04	0.1016304E-01	0.8262294E+00	0.4563408E+00
0.3543573E-01	0.1844495E+03	0.1784378E+04	0.3073971E-21	0.3520350E+00	0.3723609E+00	0.9094987E-03	0.1405430E+02	0.2500000E+00	0.9243531E+03	
0.2000000E+01	0.4613732E+03	0.2218341E-01	0.1172803E+00	0.1041563E-01	0.8433752E-01	0.3099491E+01	0.9577021E-00	0.6831360E+02	0.6999421E+00	0.3008579E+00
0.1400294E+00	0.6451439E-03	0.9565915E-01	0.8267670E-20	0.4528826E+00	0.6119055E+00	0.1008298E-01	0.2476188E+01	0.1054230E+02	0.2218666E+01	0.1874292E+01
0.1236779E+04	0.5737167E+04	0.2536528E+04	0.8143322E-03	0.2291435E+03	0.9677021E+00	0.2452698E-00	0.2465958E+02	0.2641943E+01	0.1379345E+02	0.1090018E+04
0.1453440E+03	0.1241774E+01	0.1395159E+02	0.1059132E-24	0.1481141E+03	0.1395880E+00	0.1613163E-00	0.3330835E+04	0.1016612E-01	0.8260345E+00	0.4562472E+00
0.3543439E-01	0.1844667E+03	0.1784482E+04	0.3074448E-21	0.3533498E+00	0.3728800E+00	0.9082494E-03	0.1444690E+02	0.2500000E+00	0.9224963E+03	
0.8000000E+01	0.4600053E+03	0.2207719E-01	0.1175302E+00	0.1041533E-01	0.8301279E-01	0.3099370E+01	0.9579784E-00	0.6817444E+02	0.6981200E+00	0.3018712E+00
0.1403023E+00	0.6451439E-03	0.9555231E-01	0.8279691E-20	0.4549095E+00	0.6137190E+00	0.1002775E-01	0.2479023E+01	0.1050873E+02	0.2218414E+01	0.1876847E+01
0.1235823E+04	0.5731581E+04	0.2538222E+04	0.8150410E-03	0.2293725E+03	0.9679784E+00	0.2452568E-00	0.2465828E+02	0.2646994E+01	0.1379833E+02	0.1090079E+04
0.1452855E+03	0.1241746E+01	0.1394835E+02	0.1059093E-24	0.1480631E+03	0.1396212E+00	0.1613385E-00	0.3337357E+04	0.1018602E-01	0.8247304E+00	0.4555861E+00

Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10}A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	0.1413 ± 0.0011	0.1427 ± 0.0014	0.1422 ± 0.0013	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8 \Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8 \Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
z_{drag}	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
r_{drag}	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_D	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
z_{eq}	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{s,eq}$	0.4502 ± 0.0047	0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023
f_{2000}^{143}	29.9 ± 2.9	30.4 ± 2.9	30.3 ± 2.8	29.5 ± 2.7	30.2 ± 2.7	30.0 ± 2.7
$f_{2000}^{143\times217}$	32.4 ± 2.1	32.8 ± 2.1	32.7 ± 2.0	32.2 ± 1.9	32.8 ± 1.9	32.6 ± 1.9
f_{2000}^{217}	106.0 ± 2.0	106.3 ± 2.0	106.2 ± 2.0	105.8 ± 1.9	106.2 ± 1.9	106.1 ± 1.8


PlanckTT
PlanckTT+Pol
B_{tp}
0.048
0.044
0.040
0.036
0.032
0.028
0.024
0.020
0.016
D_{tp}
0.30
0.27
$\ln(10^{10}A_s)$
3.28
3.20
3.12
3.04
Ω_m
0.27
0.26
0.25
0.24
0.01 0.02 0.03 0.04
0.27 0.30
3.04 3.12 3.20 3.28
0.24 0.25 0.26 0.27

Parameters

- In CosmoMC there are three types of parameters:
  - **Cosmological** parameters: these are the ones you want to vary, as part of the cosmological model you choose. The default is the 6 LCDM base parameters.
  - **Nuisance** parameters: these are forced to vary when you include some particular dataset because this is the way to marginalize over systematic effects. They are added automatically when you use that data.
  - **Derived** parameters: these are not varied themselves, but they depend on cosmological parameters, so you indirectly obtain constraints on them.

Convergence: Gelman-Rubin criterion

- The **R-1 estimator** is defined roughly as “the variance of the chain means divided by the mean of the variance”. A set of chains is declared **converged** when this estimator is **small** enough (typically 0.01)

GetDist products

- **root.likestats** and **root.margestats**: parameter constraints
- **root.corr** and **root.covmat**: these are the correlation matrix R_{ij} and the covariance matrix C_{ij} of the chain parameters, where $R_{ij}= C_{ij}/\mathrm{sqrt}(C_{ii}\ C_{jj})$. The file root.covmat can be used in params.ini to help in faster convergence
- **root.converge**: Contains information on the convergence of your chains

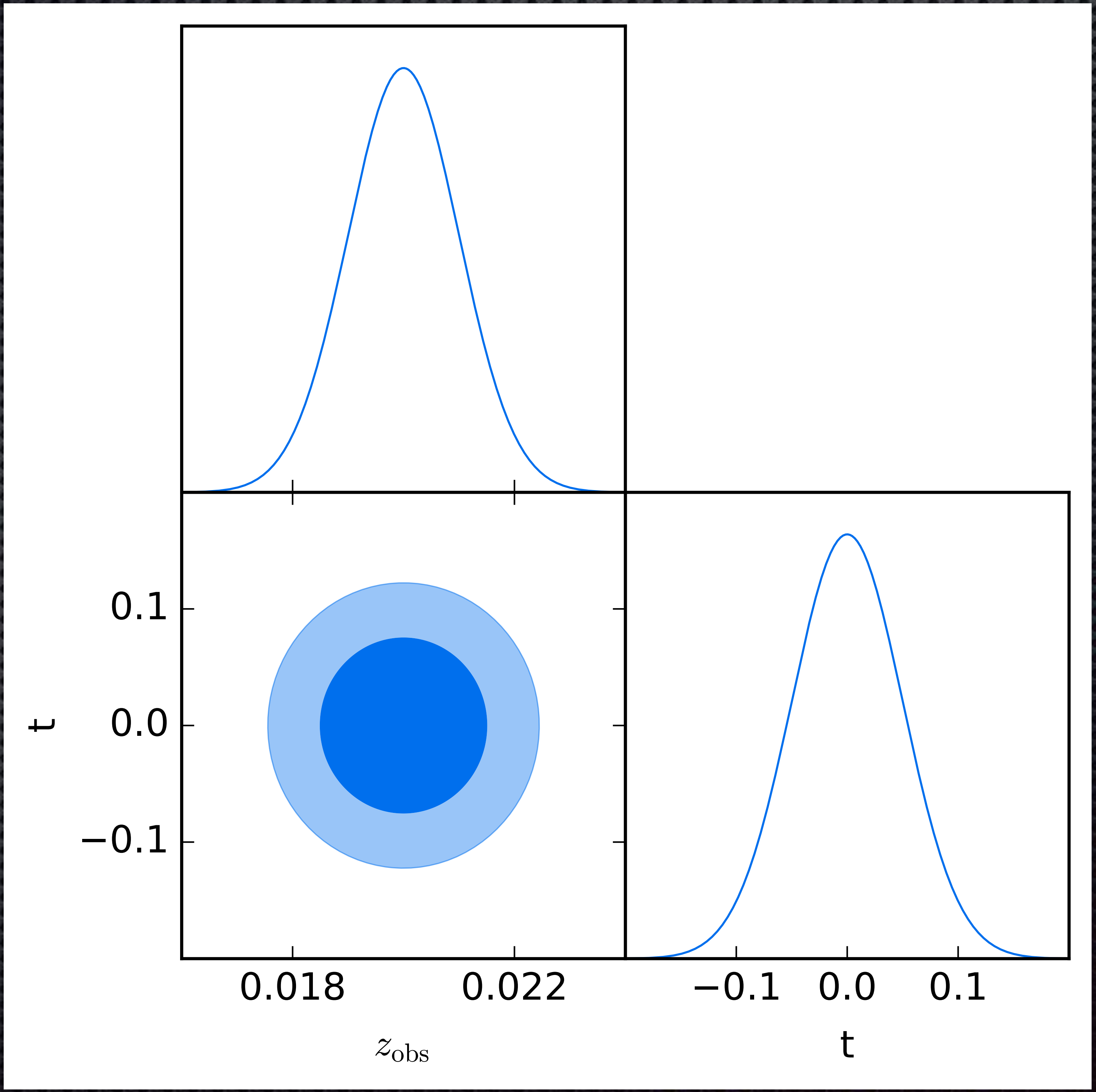
Margestats

- This is the **most valuable** product of your chain. It contains the **mean** value of each paramter, its **standard deviation** , its **lower** and **upper limits** for 68%, 95% , 99% confidence levels and **chi2**
-

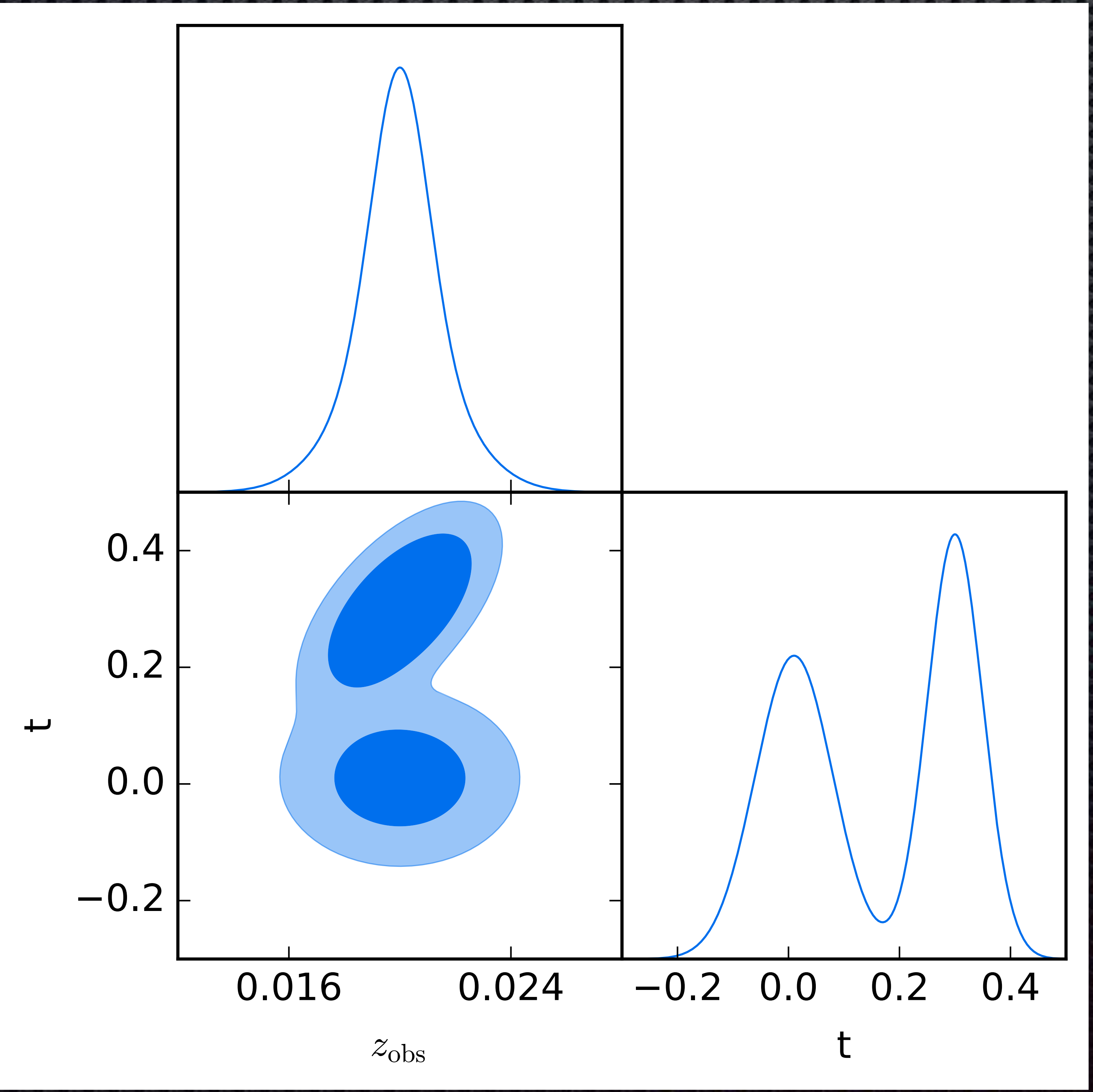
Likestats

- This file contains less relevant, but still sometimes useful information. it is NOT what you usually would report in a paper
- This contains the **best fit** value and the **bounds** of the **mean likelihood** distribution (not the posterior!) for each parameter
- Bestfit value is better computed by **action=2** anyway

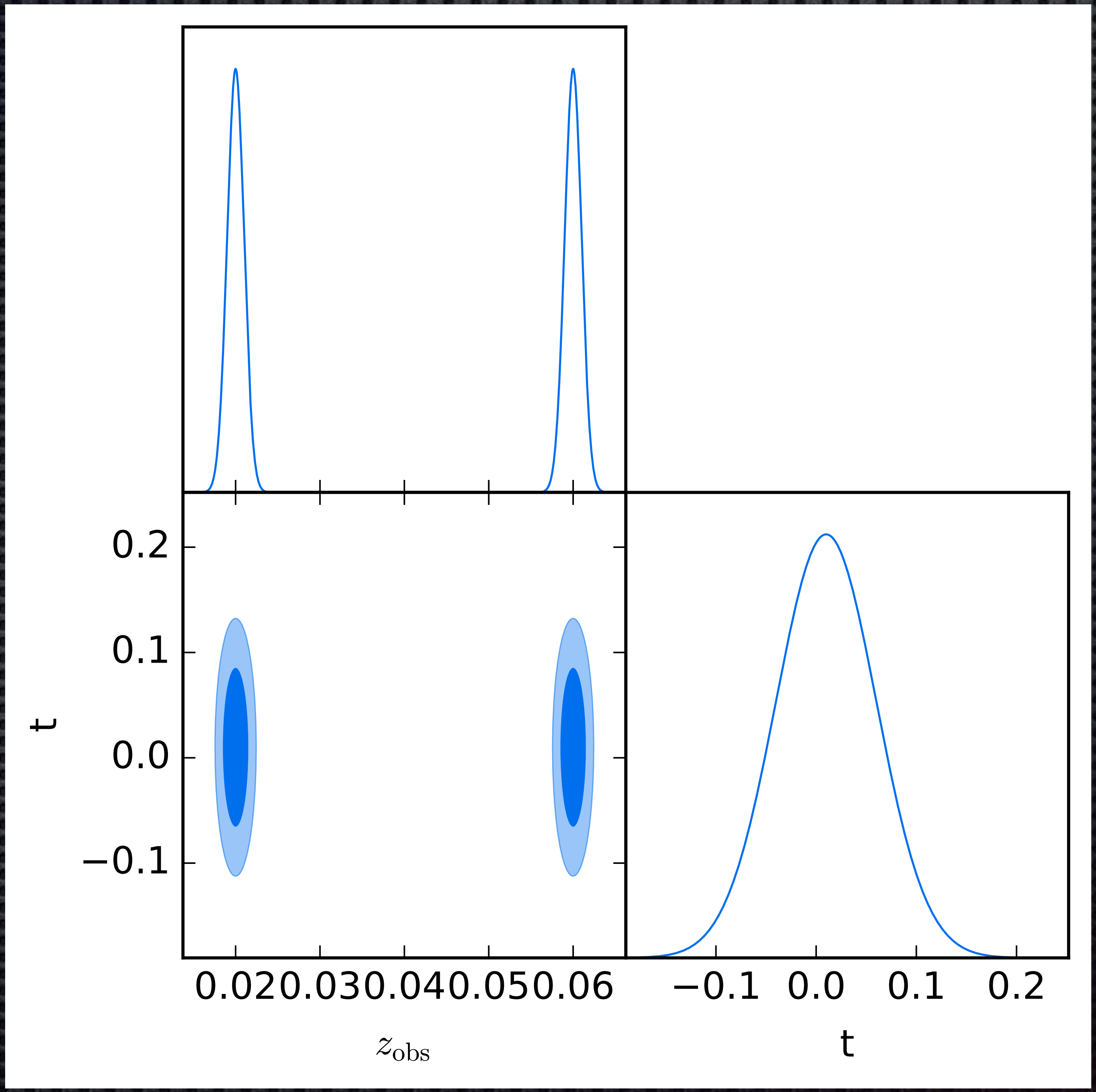
Gaussian



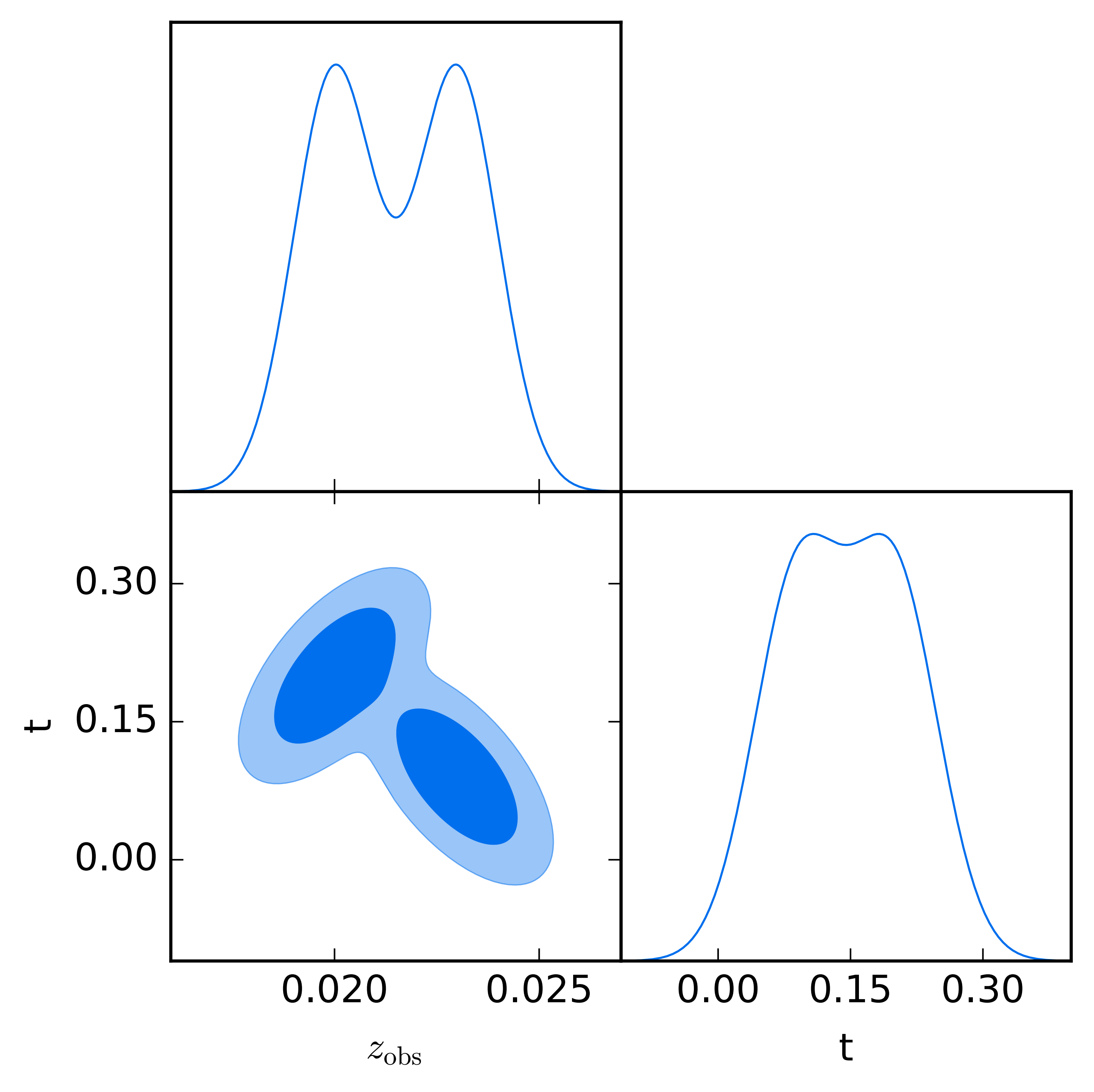
Bimodal



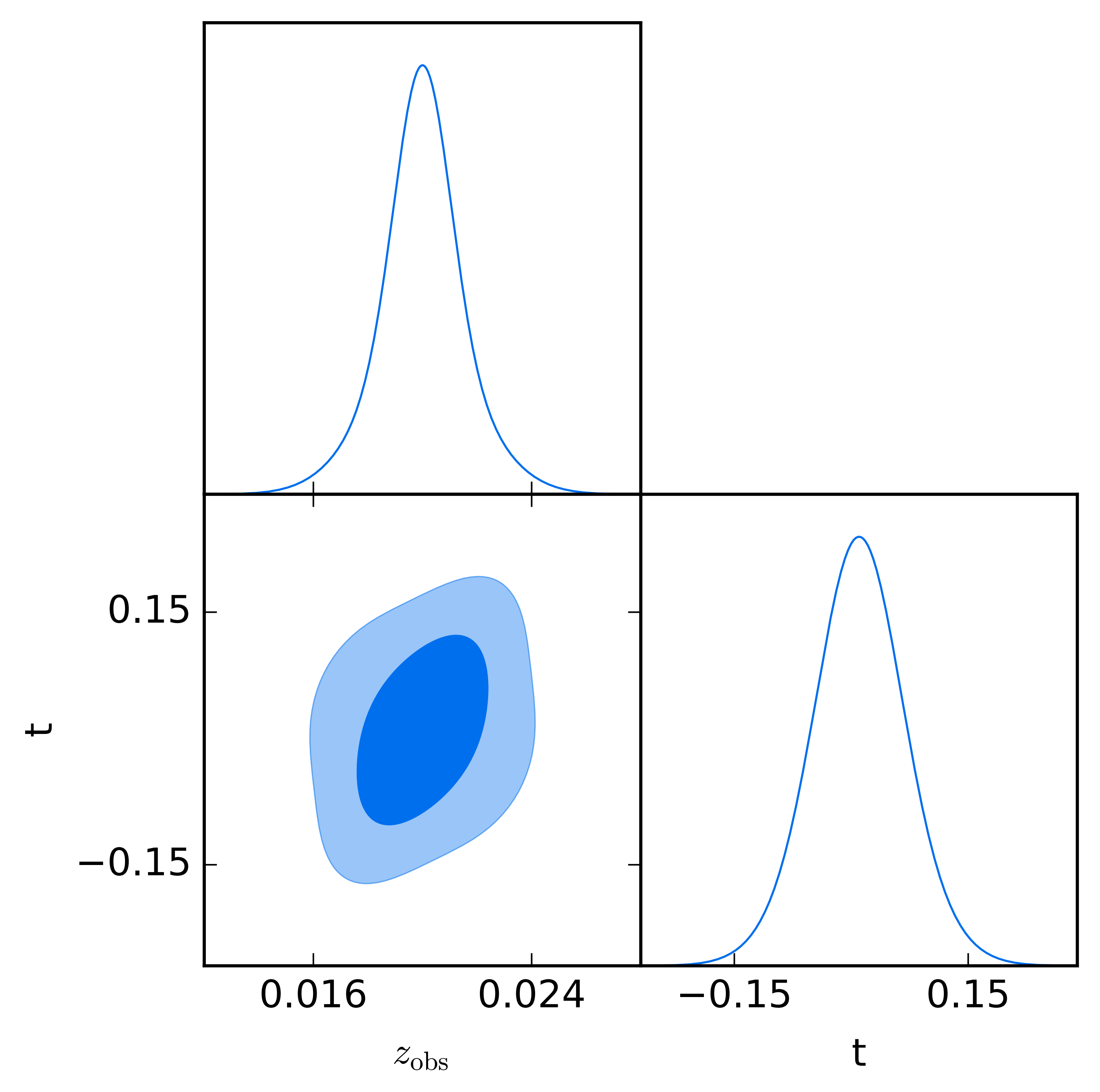
Bimodal



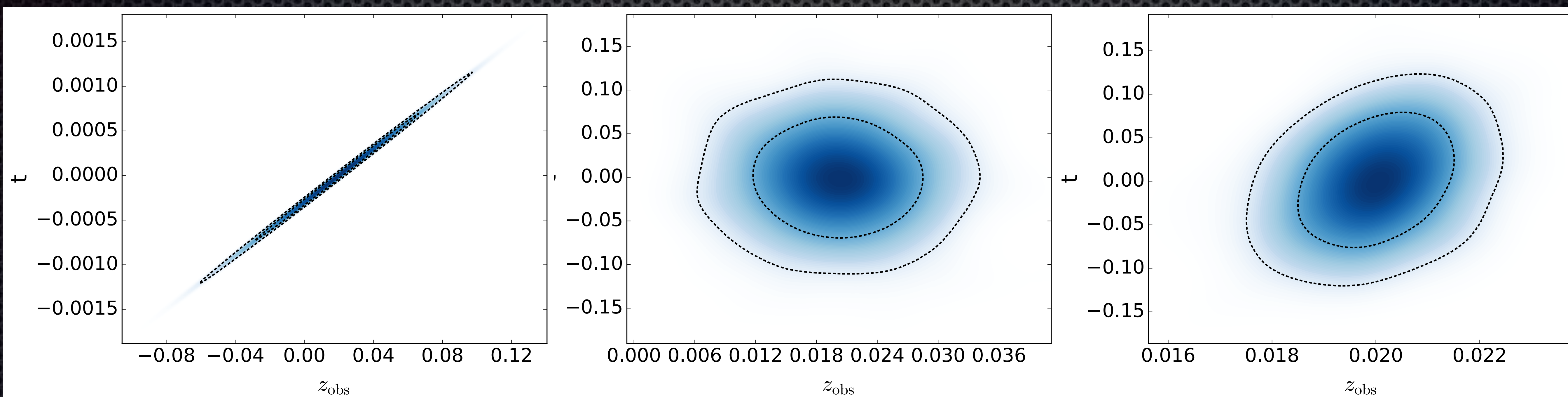
Hammer



Broad tail



Skew



t
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−0.08 −0.04 0.00 0.04 0.08 0.12
$z_{\rm obs}$
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.000 0.006 0.012 0.018 0.024 0.030 0.036
$z_{\rm obs}$
t
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.016 0.018 0.020 0.022
$z_{\rm obs}$

C=1 C=0 C=0.4

Other codes

- A strong competitor is **MontePython**, a python code that interfaces the Boltzmann solver **CLASS**. The advantage of CLASS over CAMB is that it matches the notation in Ma & Bertschinger, so it is easier to generalize
- A rising code is **cosmosis**, another python code that is extremely modular, which makes it easy to switch between MCMC samplers, Boltzmann codes
- Other samplers have been used in the past (CosmoPMC, PICO, etc.) but they do not seem to be in active development or use approximated methods