Date: 1400/10/09

In the name of God

Department of Physics
Shahid Beheshti University

OPTIMIZATION AND COMPUTATIONAL APPROACHES

Second Mid-Term exam

(Time allowed: 3 hours)

Theoretical part:

1. Finding the best fit parameter analytically: Assume a theoretical model given by $y_{th} = ax^H$. Here the model free parameters are $\{\Theta\} : \{a, H\}$. Considering the satisfaction of central limit theorem and the $(\{D\} = \{(x_i, y_i)\}, \quad i = 1, .., N)$. The covariance matrix also reads as $C_y \equiv \langle \delta y \bigotimes \delta y \rangle$.

(a) Now suppose that we have a map as $y \to z = \ln(y)$. Given the general form for covariance matrix as $C_z \equiv \langle \delta z \bigotimes \delta z \rangle$, determine the $\{a_{best}, H_{best}\}$. (5 points)

(b) Find the relation between $C_z \equiv \langle \delta z \bigotimes \delta z \rangle$ and $C_y \equiv \langle \delta y \bigotimes \delta y \rangle$.(5 points)

(c) Show that if we have diagonal covariance matrix for data, the Fisher information matrices of y_{th} and $z_{th} = \ln(y_{th})$ are similar. (5 points)

2. Fisher information matrix: for a quantum paramagnetic system in an external constant magnetic field $(\vec{B}_{ext} = B_{ext}\hat{k})$, we have the average of magnetization as: (15 points)

$$M_z = g\mu_B J \left[\left(1 + \frac{1}{2J}\right) \coth \left[\left(1 + \frac{1}{2J}\right) x \right] - \frac{1}{2J} \coth \left[\frac{x}{2J} \right] \right]$$

here $x \equiv \beta g \mu_b J B_{ext}$, $\{\Theta\} : \{g, J\}$, $\{D\} : \{(B_{ext}, M_z)_i\}, \quad i = 1, ..., N$.

Computational part:

3. Using file which is called *"sin.txt"* and consider $y_{th} = a \sin(\omega t + \phi)$. The first column of the *"sin.txt"* is "t", the second column is "y" and the last column is σ_y.

(a) Compute $\{\Theta\} : \{a, \omega, \phi\}$. Using MCMC method compute a, ω and ϕ. (15 points)

(b) According to the Fisher information matrix and taking into account the *"Cov11.txt"* data, determine the contours for 1σ, 2σ, 3σ for each pairs of parameter. (15 points)

Good luck, Movahed