Date: 1400/09/04

In the name of God

Department of Physics
Shahid Beheshti University

OPTIMIZATION AND COMPUTATIONAL APPROACHES

Mid-Term exam

(Time allowed: 3 hours)

NOTE: All question must be answered. Please write the answer of each question in separate sheet.

1. Suppose that x is a random variable and has the following distribution, $p(x) = \frac{x}{2}$ for $0 \leq x \leq 2$. Now for each of mapping from x to y, find the probability density function of y (10 points).

(a) $y = 1 - \frac{\sqrt{4-x^2}}{2}$

(b) $y = 1$

2. Show that the following conditional density function satisfies the Chapman-Kolmogorov (C-K) equation (10 points).

Hint: C-K eq is: $p(x_3, t_3|x_1, t_1) = \int dx_2 p(x_3, t_3|x_2, t_2)p(x_2, t_2|x_1, t_1)$

$$p(x_2, t_2|x_1, t_1) = \frac{1}{\sqrt{2\pi(t_2 - t_1)}} \exp \frac{-(x_2 - x_1)^2}{2(t_2 - t_1)}$$

and $t_1 < t_2 < t_3$ also $\tau \equiv t_3 - t_2 = t_2 - t_1$.

3. A particle leaves the origin under the influence of the force of gravity and its initial velocity v_0 forms an angle ϕ with the horizontal axis. The path of the particle reaches the ground at a distance $d_{max} = \frac{v_0^2}{g} \sin(2\phi)$ from the origin. Assuming that ϕ is a random variable uniform between 0 and $\pi/2$, determine (10 points):

(a) The probability density of d,

(b) The probability that $d < d_{max}$.

4. Consider a 2-Dimensional 5×5 square lattice magnetic system with the following Hamiltonian:

$$H(\{S\}) = -\sum_{\langle i,j \rangle}^{N=5} J_{ij} s_i s_j$$

in which J_{ij} has distribution $P(J) = \frac{1}{\sqrt{2\pi}} \exp(\frac{-J^2}{2})$ and s_i can take the values $\{1, -1\}$. Find the minimum energy using a stochastic method for sampling. To this end, run over 10 ensemble and each ensemble only includes 1000 random configurations and accordingly, compute the $\langle H \rangle$ and corresponding mean-standard deviation (10 points).

5. For the input data set:

(a) Find the probability density function of mentioned data and also its peaks and plot them (5 points).

(b) Smooth the time series using kernel $K(x, x') = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-(x-x')^2}{2\sigma^2}$ for $\sigma = 0.1$ and $\sigma = 2$, separately and plot the data and smoothed series (5 points).

(c) Now, find the probability density function of smoothed data and also its peak and plot them (5 points).

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(d) Compute first three cumulants of original and smoothed series and associated peaks distribution (5 points).

(e) Discuss your results in part (d) (5 points).

Good luck, Movahed