بسم الله الرحمن الرحیم   جلسه ۲۸/۹/۱۴۰۰

Exercise Set 6 No. 2.

$$\chi(\{\theta\}) = \sum_{i=1}^{580} \frac{\left[\mu_i - \mu_{Theory}(z_i, \{\theta\})\right]^2}{\sigma_{\mu_i}^2} \qquad i=1 \dots 580$$

we should use the covariance matrix for observed data.

$$\underset{1\times1}{\chi^2(\theta)} = \underbrace{\left[\mu_{obs} - \mu_{The}\right]^T}_{(1\times N)} \cdot \underset{N\times N}{C^{-1}} \cdot \underset{N\times1}{\left[\mu_{obs} - \mu_{The}\right]}$$

$cov = V \longrightarrow$

$$\chi^2(\{\theta\}) = \underset{1\times N}{\left(\mu_{obs}(z_1) - \mu_{Th}(z_1, \{\theta\}), \mu_{obs}(z_2) - \mu_{Th}(z_2, \{\theta\}) \dots\right)} \cdot \underbrace{\left(\; C^{-1} \;\right)}_{N\times N} \underset{N\times 1}{\begin{pmatrix} \mu_{obs}(z_1) - \mu_{Th}(z_1, \{\theta\}) \\ \mu_{obs}(z_2) - \mu_{Th}(z_2, \{\theta\}) \\ \vdots \\ \vdots \\ \mu_{obs}(z_N) - \mu_{Th}(z_N, \{\theta\}) \end{pmatrix}}$$

یک بار ابتدا
در برنامه اول
معکوس را حساب می‌کنیم

MCMC algorithm for SNIa.

Import Data set $\longrightarrow$ $N = 580$, $\boxed{M = 10^6}$

$\chi^2_{old} = 0$

Construct the $A \equiv C^{-1}$, $\chi^2_{min} = 1000000000$

Select a typical set for model free parameters

$$\boxed{\{\theta\}_{old} = \{\Omega_{m\,old} = 0.2,\ \Omega_{\lambda\,old} = 0.5,\ \omega_{old} = -2,\ H_0 = 70\}}$$

Suppose that the Covariance Matrix is diagonal.

to Compute the χ^2_{old} we have

loop $i=1, N$

Call a Subroutine to Calculate d_L

$\Omega_k = 1 - \Omega_m - \Omega_\lambda$

$$d_L = \frac{(1+z_i)}{\sqrt{|\Omega_k|}} \sinh\left[\sqrt{|\Omega_k|}\int_0^{z_i} \frac{dz'}{\left[\Omega_m(1+z')^3 + \Omega_\lambda(1+z')^{3(\omega+1)} - \Omega_k(1+z')^2\right]}\right]$$

$$\mu_{ther} = 5\log_{10} d_L + 5\log_{10}\frac{3\times10^5}{H_0} + 25$$

$$\chi^2_{old} = \chi^2_{old} + \frac{[\mu_i - \mu_{ther}]^2}{\sigma_i^2}$$

Diagonal Element of diagonal C

$$e^{-\left[\frac{\chi^2}{2}\right]} \quad e^{-\left(\frac{\ \ }{2}\right)^2}$$

Endloop (Compute χ^2_{old})

Using prior $\bar{\chi}^2_{old} = \chi^2_{old} + \frac{(\Omega_{m\,old} - 0.3)^2}{2(0.1)^2} + \frac{(\Omega_{\lambda\,old} - 0.7)^2}{2(0.1)^2} + \frac{(\omega_{old}+1)^2}{2(1)^2}$

loop on MCMC $j = 1, M$

Select $\{\theta\}_{New}$ from $\{\theta\}_{old}$

New Parameter set

$$\left\{\begin{array}{l} \Omega_{m New} = \Omega_{m old} + \Delta\Omega_m \\ \Omega_{\lambda New} = \Omega_{\lambda old} + \Delta\Omega_\lambda \\ \omega_{New} = \omega_{old} + \Delta\omega \\ H_{0 New} = H_{0 old} + \Delta H_0 \end{array}\right.$$

Box Muller

$\chi^2_{New} = 0$

Loop $i = 1, N = 580$

Compute χ^2

Call d_L calculator

$\mu_{the} = \checkmark$

$$\chi^2_{New} = \chi^2_{New} + \frac{[\mu_{obs}(i) - \mu_{Th}]^2}{\sigma_i^2}$$

End loop

$\chi^2_{New}(\{\theta\}_{New})$

with prior

$$\chi^2_{New} = \chi^2_{New_2} + \frac{(\Omega_{m New} - 0.3)^2}{2(0.1)^2} + \frac{(\Omega_{\lambda New} - 0.7)^2}{2(0.1)^2} + \frac{(\omega_{New} + 1)^2}{2}$$

$$\Delta\chi^2 = \chi_{New}(\{\theta\}_{New}) - \chi_{old}(\{\theta\}_{old})$$

$\xi =$ Call Random Number

Metropolis part

if $\xi \leq e^{-(\Delta\chi^2/2)}$ Then

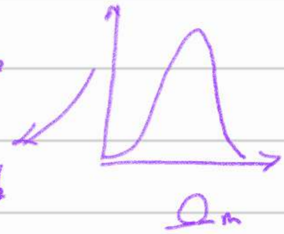
$\{\theta\}_{old} = \{\theta\}_{New}$

$\chi^2_{old} = \chi^2_{New}$

End if

$\{\Omega_m, \Omega_\Lambda, w, H_0\}, \bar{\chi}^2_{old}$

* Write $\{\Theta\}_{old}$, χ^2_{old} * $\longrightarrow$ Likelihood



find the min of χ^2_{old}

if $\chi^2_{old} \leqslant \chi^2_{min}$

Then $\chi^2_{min} = \chi^2_{old}$

$\{\Theta\}_{best} = \{\Theta\}_{old}$

End if

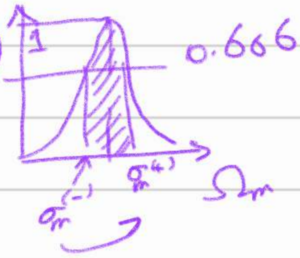
$\Omega_m \longrightarrow \min\{\bar{\chi}^2_{old}\}$.

$= e^{-\frac{(\min\{\bar{\chi}^2_{old}\} - \bar{\chi}^2_{min})}{2}}$

Local

Global

Relative posterior



End loop MCMC

Global

$\boxed{\text{Print } \{\Theta\}_{best}, \chi^2_{min}}$

You may find different values for different Run

End program

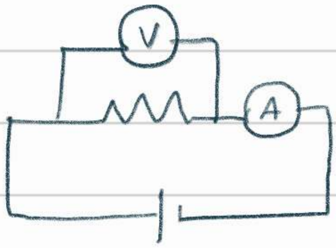
Convergence check

rule of thumb

from physical points of view

$$\Omega_m = \frac{\text{Energy Density}}{\text{Relative}} \longrightarrow \cancel{X} \quad O(1)$$

Example-



I	V
I_1	V_1
I_2	V_2
I_N	V_N

$$C_{ov} = \begin{bmatrix} \langle \delta V_1 \delta V_1 \rangle & \langle \delta V_1 \delta V_2 \rangle & \cdots & \langle \delta V_1 \delta V_N \rangle \\ \langle \delta V_2 \delta V_1 \rangle & \langle \delta V_2 \delta V_2 \rangle & \cdots & \\ \vdots & & & \\ \langle \delta V_N \delta V_1 \rangle & & & \langle \delta V_N \delta V_N \rangle \end{bmatrix}_{N\times N}$$

Statn

$$\sigma_{11}^2 = \langle \delta V_1 \delta V_1 \rangle = \langle (V_1 - \langle V_1 \rangle)^2 \rangle_{ens}$$

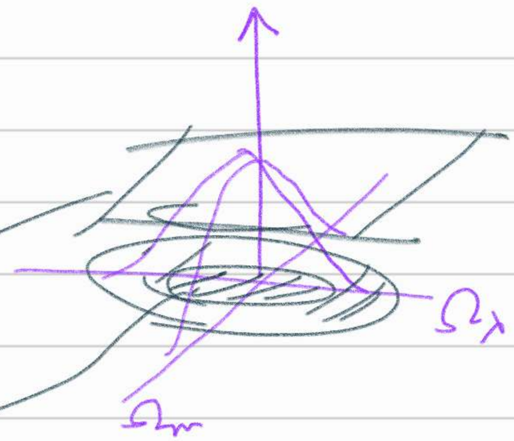
$$\sigma_{12}^2 = \langle \delta V_1 \delta V_2 \rangle_{en} = \langle \underbrace{(V_1 - \langle V_1 \rangle)}_{+} \underbrace{(V_2 - \langle V_2 \rangle)}_{-} \rangle$$

$$\begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1N}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & & \sigma_{2N}^2 \\ & & \ddots & \\ \sigma_{N1}^2 & & & \sigma_{NN}^2 \end{bmatrix}_{N\times N}$$

$$\{\Omega_m, \Omega_\lambda, \omega, H_0, \bar{\chi}^2\}$$

$$e^{-\frac{(\min\{\chi^2\} - \bar{\chi}^2_{\min})}{2}}$$

$$\Omega_m, \Omega_\lambda \longrightarrow \min\{\bar{\chi}^2\}$$



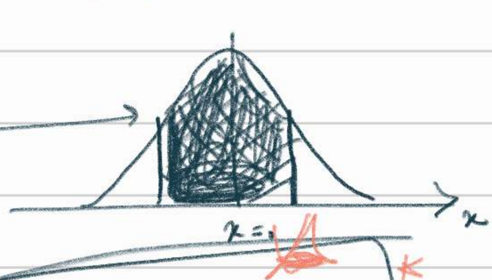
$$e^{-\frac{2.3}{2}}$$

1σ

e

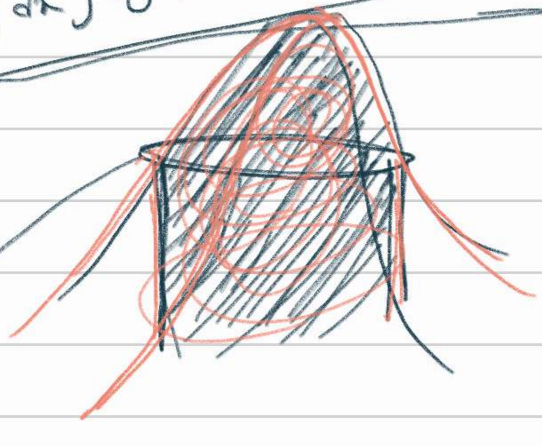
$$68.3\% = \int_{-1\sigma}^{+1\sigma} p(x)\, dx$$

$\langle x \rangle = 0$



$$\int dx \int dy \; p(x,y) = 1 \rightarrow \text{Normalized}$$

$$68.3\% = \int_{-1\sigma_1}^{+1\sigma_1} dx \int_{-1\sigma_2}^{+1\sigma_2} dy \; p(x,y)$$

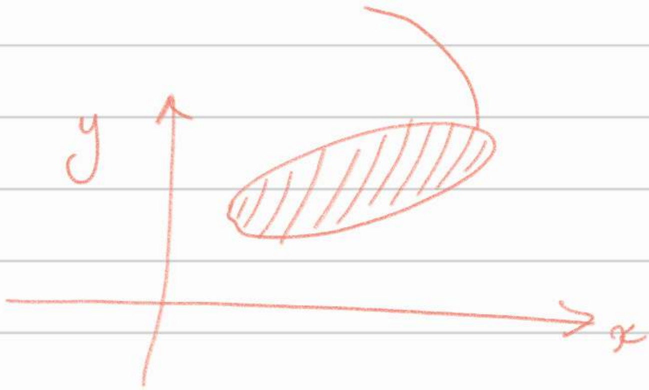


Δ همان بیضی‌گون را که رسم کردیم

گونی بود.

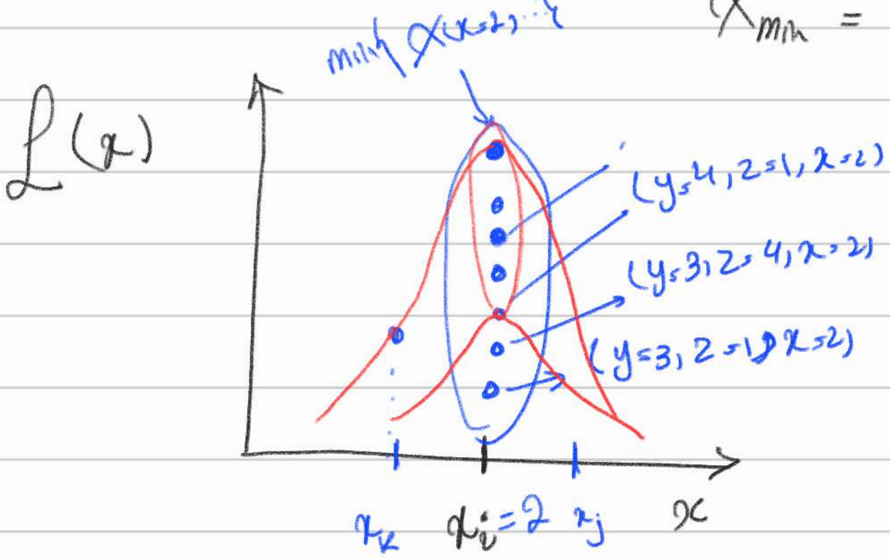
$$\frac{(x-\bar{x})^2}{\sigma_x^2} + \frac{(y-\bar{y})^2}{\sigma_y^2} = \Delta\chi^2$$

$$1\sigma \leftarrow 2.73 \leftarrow \Delta\chi^2 = \frac{(x-\bar{x})^2}{\sigma_x^2(1-\rho^2)} + \frac{(y-\bar{y})^2}{\sigma_y^2(1-\rho^2)} - \frac{2\rho}{(1-\rho^2)} \frac{(x-\bar{x})(y-\bar{y})}{\sigma_x^2 \; \sigma_y^2}$$



$$\{x, y, z\} \longrightarrow \underbrace{\{x_{best}, y_{best}, z_{best}\}}.$$

$$\chi^2_{min} = \text{Global min of } \chi^2$$



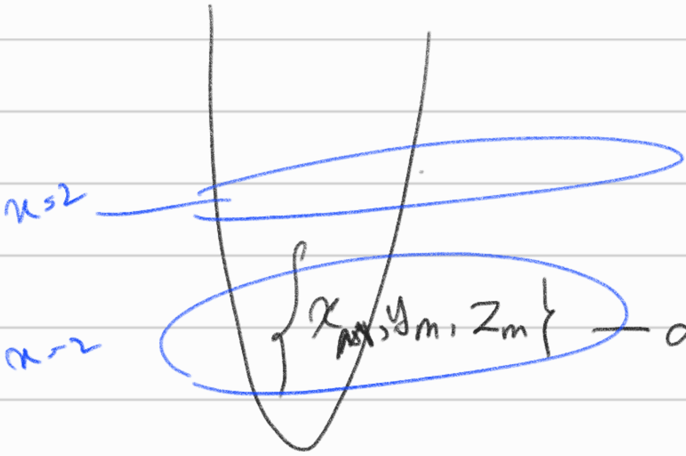
$$\mathcal{L}(x) = \underbrace{\int dy\, dz\, \mathcal{L}(x,y,z)}_{\mathcal{L}}$$

$\{x_1, \underline{y_1}, \underline{z_1}\} - \mathcal{L}(x_1, y_1, z_1)$ or $\underline{\chi^2(x_1, y_1, z_1)}$

$\chi^2(x=2, y_1, z_1) \leftarrow x=2$

$\{x_2, y_2, z_2\} - \mathcal{L}(x_2, y_2, z_2)$ or $\chi^2(x_2, y_2, z_2)$

$\chi^2(x=2, y_2, z_2) \leftarrow x=2$



$\{x_m, y_m, z_m\}$ — $\mathcal{L}(x_m, y_m, z_m)$, $\chi^2(x_m, y_m, z_m)$