بسم الله الرحمن الرحیم

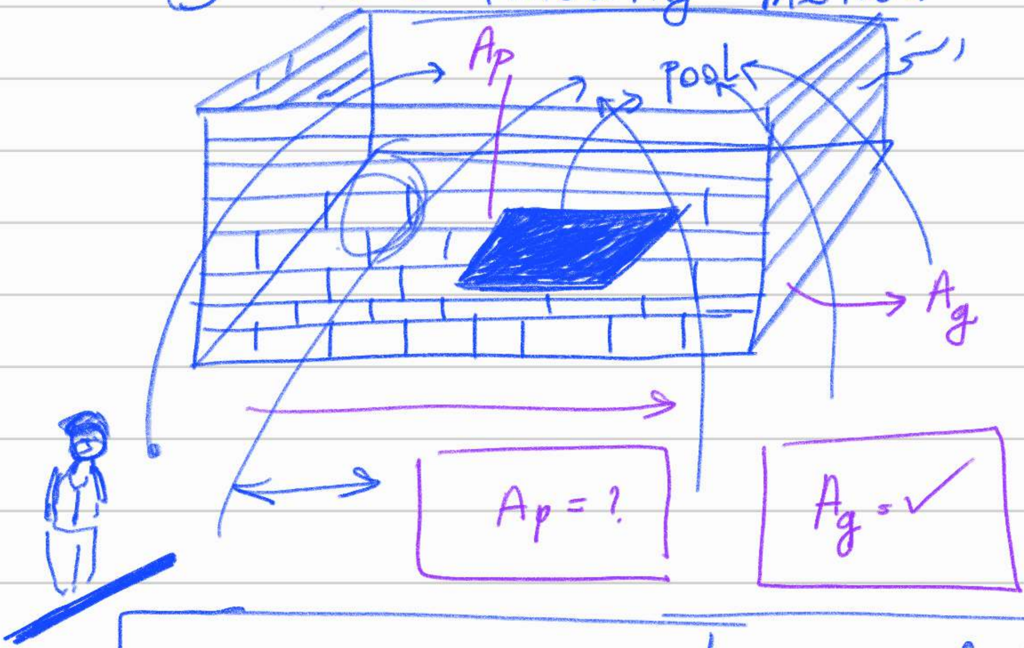
جلسه ۱۴/۹/۱۴۰۰

* Numerical Integration : Stochastic approach

"بهره گیری می‌کند از اعداد تصادفی" = Using Random generator to solve e.g. a typical Integral Numerically.

① Stone throwing method

روش پرتاب سنگ



A_g = The area of Garden

A_p = Area of Pool

$$\frac{A_p}{A_g} \stackrel{?}{=} \frac{N_p}{N_g} =$$

N_p → The Number of Stone throwing inside the pool

N_g → $\boxed{\text{The Number of total Stone}}$

So our Strategy to throw the stone should be as follow

① we should throw the stones With Random direction, Random speed

* Random Angle $N_g =$ ✓

↳ ② the person must walk around the garden

* We finally ensure that we could *

* Scan all part of Garden *

Shalap Sound ☺ (Splash)

$$\frac{A_p}{A_g} = \lim_{N_g \to \infty} \frac{N_p}{N_g}$$

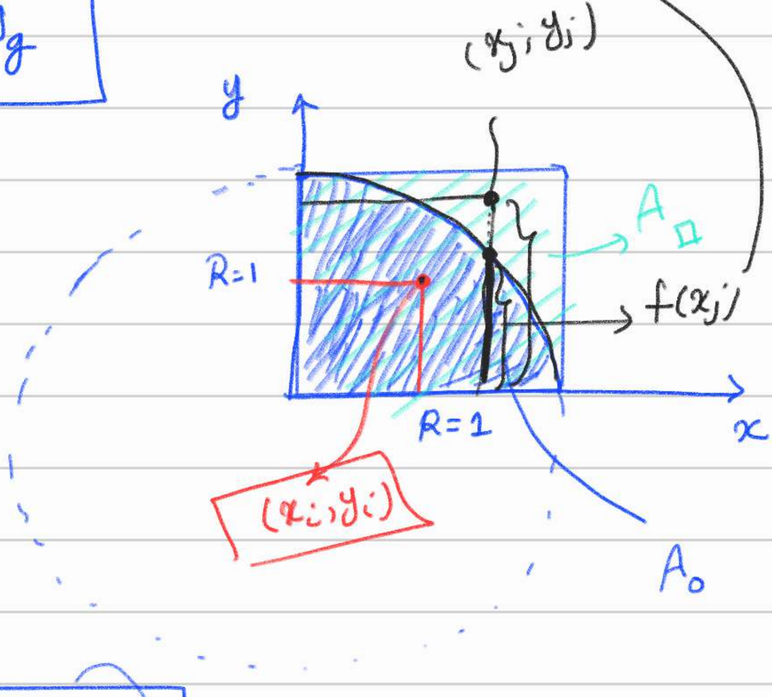
N_p → Shalap Sound (Splash)

N_g → total No of stone.

$$\boxed{A_p = \lim_{N_g \to \infty} A_g \frac{N_p}{N_g}}$$

$f(x_j) \equiv \sqrt{R^2 - x_j^2}$

$f(x_j) = \sqrt{1 - x_j^2}$



Ex: $\pi = ?$

$$A_o = \frac{\pi R^2}{4} = \frac{\pi}{4}$$

$$A_{\square} = R^2 = 1$$

$$\frac{A_o}{A_{\square}} = \frac{\pi}{4} \rightarrow \boxed{\pi = 4 \frac{A_o}{A_{\square}}}$$

$$\frac{A_o}{A_{\square}} = ?$$

$$\frac{N_o}{N_{\square}}$$

- Stone throwing is Generate a pair of (x, y) [location of Stone] by using Random generator

$$\left[\begin{array}{c} 0 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{array}\right]$$

- Shalap sound $\longrightarrow$ Check a Condition

$$\boxed{y_j \leq \sqrt{1 - x_j^2}}$$

Numerical Algorithm for Stone throwing

loop $N_{\square} = N_{min}, N_{Max}$

$N_{total} = N_{\square} \doteq$ تعداد کل سنگ‌های پرتابی

- $N_0 = 0$

loop on $i = 1, N_{\square}$

$a = 0$
$b = 1$

① , ② Generate two Random Number (x_i, y_i)

$$\begin{cases} x_i = a + r_1 (b - a) \\ y_i = a + r_2 (b - a) \end{cases}$$

$x \in [a, b] \quad r_1 \in [0, 1]$

$y \in [a, b] \quad r_2 \in [0, 1]$

③ if $y_i \leq \sqrt{1 - x_i^2}$

$N_0 = N_0 + 1$

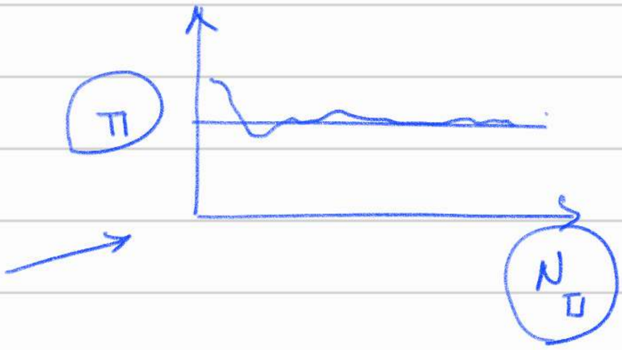
Endif

End loop

$$Pi = 4 \times \frac{N_o}{N_\square} \qquad A_\square = 1$$

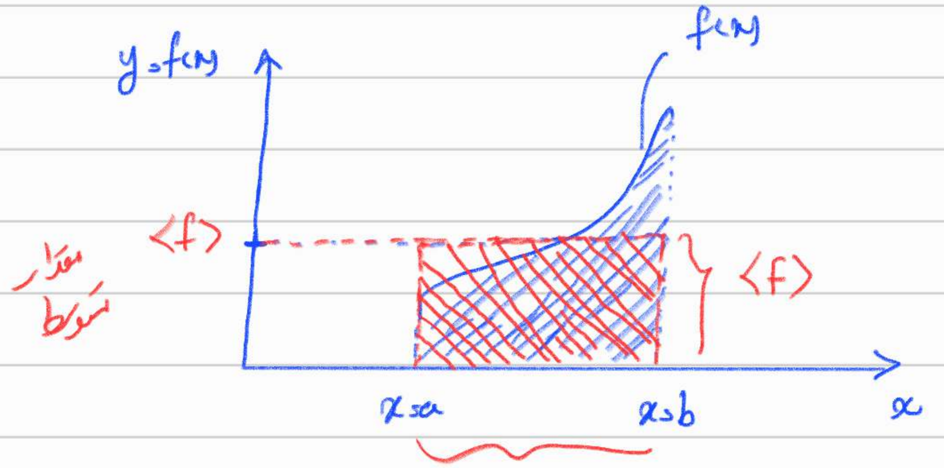
Write $N_\square$, Pi

→ End loop



② Mean Value.

$$\frac{\text{تولید دوتا عدد تصادفی + بررسی یک شرط}}{\text{گلوگاه}}$$



$$A^{?} = \int_{x=a}^{x=b} dx\, f(x) = B = \langle f \rangle \times (b-a)$$

$$\boxed{\langle f \rangle = \frac{1}{b-a} \int_a^b f(x)\, dx}$$

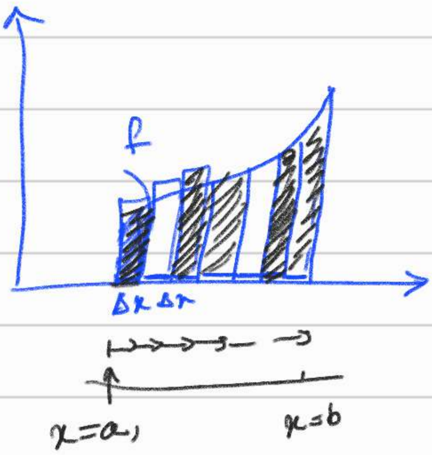
$$A = \langle f \rangle^{?} (b-a)$$

$$\langle f(x)\rangle = \frac{1}{N}\sum_{i=1}^{N} f(x_i) \checkmark$$

a standard Definition of mean Value

$$A = \frac{(b-a)}{N}\sum_{i=1}^{N} f(x_i)$$

$$\Delta x = \frac{b-a}{N}$$



$$\boxed{A = \lim_{N\to\infty}\sum_{i=1}^{N} f(x_i)\,\Delta x}$$

اعداد تصادفی با توزیع $p(x)$ PDF

Notice to the $\displaystyle A = \int_a^b dx\, f(x)\,\frac{p(x)}{p(x)}$

$$A = \left\langle \frac{f(x)}{p(x)} \right\rangle = \int_a^b dx \frac{f(x)}{p(x)}\, p(x)$$

$$A = \left\langle \frac{f(x)}{p(x)} \right\rangle_x \longrightarrow \boxed{\text{with PDF of } p(x)}$$

If Suppose that the x is Random Number with cts PDF betwen $x \in [a,b]$

$$\int_a^b dx\, \underset{\sim}{\overset{\text{cts}}{p(x)}} = 1 \qquad p\cancel{(x)}(b-a) = 1 \longrightarrow \boxed{p = \frac{1}{(b-a)}}$$

$$A = \left\langle \frac{f(x)}{p(x)} \right\rangle = \left\langle \frac{f(x)}{\frac{1}{(b-a)}} \right\rangle_x$$

$$= \frac{1}{\frac{1}{(b-a)}} \underbrace{\langle f(x) \rangle_x}$$

اعداد تصادفی که با احتمال یک

بین a و b تولید می‌کنه

$$= \frac{(b-a)}{N} \sum_{i=1}^{N} f(x_i)$$

$$\boxed{A = \Delta x \sum_{i=1}^{N} f(x_i)}$$

$(x, \cancel{y})$ + $\underbrace{\text{condition}}$ $\longrightarrow$ $\bar{\bar{x}}$

Mean Value

$$A = \int_a^b f(x)\, dx \longrightarrow A = \left\langle \frac{f(x)}{\underbrace{p(x)}} \right\rangle \quad x \in [a, b]$$

با این تابع توزیع [illegible]

$$\boxed{A = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}}$$

تولید عدد تصادفی

در بازه

$[a, b]$

با تابع توزیع $p(x)$

نمونه برداری با اهمیت

(3) Importance Sampling

$P(x) = ?$

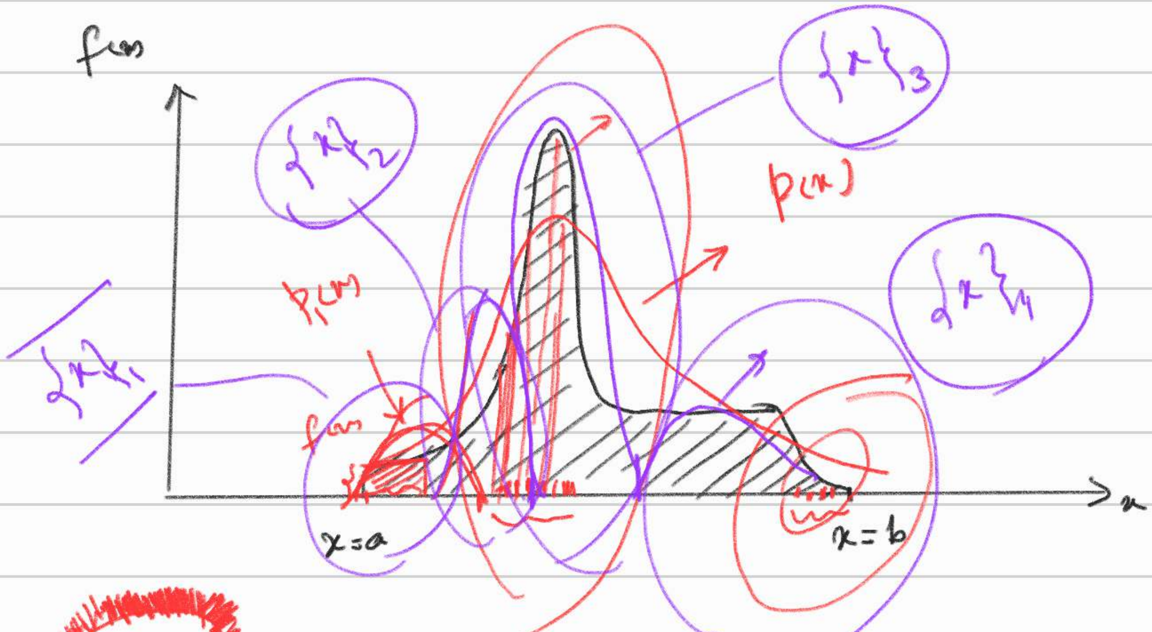
ایده این است که مقادیری از x که نقش غالب در تعیین دقت

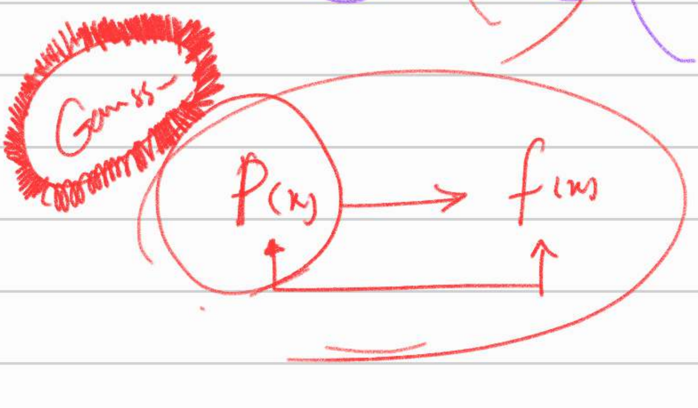
the value of which has

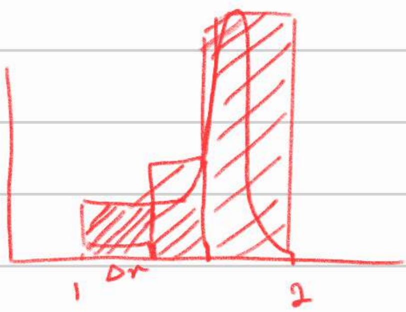
Dominant contribution to determine the Accuracy of our computational approach must be generated with higher probability.

$$A = \int_a^b dx\, f(x) = \int_a^b dx\, \frac{f(x)\, p(x)}{p(x)} = \left\langle \frac{f(x)}{p(x)} \right\rangle_x$$

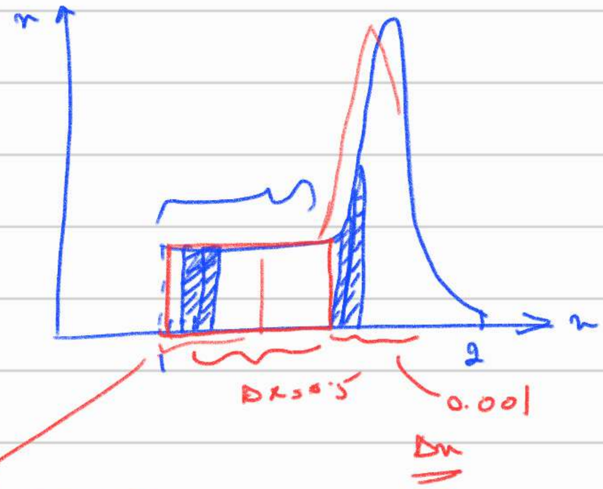
with the PDF of $p(x)$







دقت کم است $\leftarrow A = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$



$N = 1000$

$\Delta x = \frac{2-1}{1000} = \frac{1}{1000}$

$\Delta x = 0.001$

$\frac{f(x)}{p(x)}$

x

$(1,2)$

Importance Sampling

Importance Sampling $\longrightarrow$ Expectation Value

Observable Quantities