**Date: 1393/03/20**

**Name**:

In the name of God

Department of Physics
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# ADVANCED TOPICS IN STATISTICAL PHYSICS II

## Final exam

## (Time allowed: 3 hours)

**NOTE:** All question must be answered. Legibility, good hand-writing and penmanship have 5 additional marks. Please write the answer of each question in separate sheet.

**1.** Suppose a particle governed by so-called Langevin equation as follows:(20 marks)

$$\dot{v}(t) = \xi v(t) + \eta(t)$$

and $\langle \eta(t)\eta(t')\rangle = 2g\delta(t-t')$ compute:
**A:** $v(t)$
**B:** $\langle v(t)^2 \rangle$
**C** :$\langle v(t)v(t')\rangle$
**D** :$\langle x(t)x(t')\rangle$

**2.** Show the equivalence between Forward and Backward Kramers-Moyal expansion. (10 marks).

**3.** For two interacting particles in an external potential as:

$$\dot{x}_1 = v_1; \qquad \dot{v}_1 = -\gamma_1 v_1 - f_{ext}(x_1) - \frac{m_1+m_2}{m_1}\frac{\partial}{\partial x_1} f_{int}(x_1-x_2) + \Gamma_1$$

$$\dot{x}_2 = v_2; \qquad \dot{v}_2 = -\gamma_2 v_2 - f_{ext}(x_2) - \frac{m_1+m_2}{m_2}\frac{\partial}{\partial x_2} f_{int}(x_1-x_2) + \Gamma_2$$

here $\Gamma_1$ and $\Gamma_2$ are Langevin forces and $\langle \Gamma_1(t)\Gamma_1(t')\rangle = 2g_1\delta(t-t')$ and $\langle \Gamma_2(t)\Gamma_2(t')\rangle = 2g_2\delta(t-t')$. Write down Fokker-Planck equation for joint probability distribution function, $p(x_1, v_1; x_2, v_2; t)$(15 marks).

**4.** Using the transformation of variables, show that Fokker-Planck equation in the new coordinate is (20 marks):

$$\left(\frac{\partial p(x',t')}{\partial t}\right)_{x'} = \left(-\frac{\partial}{\partial x'_k} D'^{(1)}_k + \frac{\partial^2}{\partial x'_k \partial x'_r} D'^{(2)}_{kr}\right) p(x',t')$$

where:

$$D'^{(1)}_k = \left(\frac{\partial x'_k}{\partial t}\right)_x + \frac{\partial x'_k}{\partial x_i} D^{(1)}_i + \frac{\partial^2 x'_k}{\partial x_i \partial x_j} D^{(2)}_{ij}$$

$$D'^{(2)}_{kr} = \frac{\partial x'_k}{\partial x_i}\frac{\partial x'_r}{\partial x_j} D^{(2)}_{ij}$$