In the name of God

Department of Physics
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CRITICAL PHENOMENA

Exercise Set 3

(Due Date: 1401/08/20)

1. Response function: Suppose that $\mathcal{H} = \bar{\mathcal{H}}(\{S\}) + \sum_i H_i S_i$
   A: Show that $G(i,j) = \frac{1}{\beta} \frac{\partial}{\partial H_i} \left(\frac{1}{Z\beta} \frac{\partial Z}{\partial H_j} \right)$. Explain the meaning of this quantity.
   B: Write down the $\sum_{ij} G(i,j)$ using partition function.
   C: Suppose that for Homogeneous case $G(R) = \frac{1}{R^\sigma} f(R/\xi)$. Derive the susceptibility in $d-$Dimension.

2. Mean field theory: Show that $F^{\text{mean}-\text{field}} \geq F$ for Ising model. What about $T_c^{\text{mean}-\text{field}}$ and T_c.

3. Mean field theory: Incorporating the correction up to $\mathcal{O}(\delta s^3)$, derive the effective magnetic field at location of s_i.

4. Show that the coefficient of M^2 in thermodynamical free energy, $f = a + btM^2 + cM^4 + ...$ is related to correlation length scale (ξ), namely $bt \sim \xi^{-2}$. What can we result?

Good luck, Movahed

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