In the name of God

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SELECTED TOPICS COURSE

Exercise Set 3

1. For a smoothed stochastic filed expand $\langle F\rangle$ around $\langle F\rangle_G$ up to $\mathcal{O}(\sigma_0^4)$.

2. Suppose that $F \equiv \delta_D(\alpha-\nu)$. So write a perturbative expansion of $\langle F\rangle$ up to $\mathcal{O}(\sigma_0^3)$ in 1-Dimension.

3. For a given $(1+1)$-D Gaussian signal, calculate $\langle n_{up}(\nu)\rangle = \langle \delta(x-\alpha)\Theta(\eta)\eta\rangle$. Suppose that $\langle x\eta\rangle = 0$ and $\langle \eta\eta\rangle = \sigma_1$. Derived the non-Gaussian part up to $\mathcal{O}(\sigma_0^2)$.

4. Using data set (0.2.txt, 0.5.txt and 0.8.txt), compute up-crossing statistics as a function of threshold, α.

5. Bias definition: The relation between Unweighted TPCF and weighted TPCF can be considered as:

$$\Psi_{fg}(R) = \mathcal{B}_{fg}C(R)$$

(a) For Up-crossing feature determine $\mathcal{B}$ for 1+1-D stochastic field.

(b) For a sharp clipping feature, we have $f \equiv A\Theta(\alpha-\vartheta)$. Here A is a normalization coefficient. The average value of sharp clipping in a Gaussian random field can be written by:

$$\langle f(\alpha)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A\Theta(\alpha-\vartheta)e^{-\alpha^2/2}d\alpha$$

Now, we expand $\Theta(\alpha-\vartheta)$ in terms of Hermite polynomials defined by $H_n = e^{x^2/2}(d/dx)^n e^{-x^2/2}$ with $\langle H_n(x)H_m(x)\rangle\delta_{nm}m!$. Therefore we have

$$A\Theta(\alpha) = \sum_{k=0}^{\infty}\frac{J_k}{k!}H_k(\alpha)$$

Show that

$$J_k = \frac{\vartheta H_{k-1}(\vartheta)}{\vartheta\sqrt{\pi/2}\exp(\vartheta^2/2)\mathrm{erfc}(\vartheta/\sqrt{2})}$$

(c) Show $\lim_{\vartheta\to\infty} J_k = \vartheta^k$

(d) Show $\langle\Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1+\Psi(R) = \sum_{k=0}^{\infty}\frac{J_k^2}{k!}C_{\alpha\alpha}^k(R)$, where $R = |r-r'|$.

(e) For $R\to\infty$ show $\langle\Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1+\Psi(R) \sim J_1^2 C_{\alpha\alpha}(R)$. Here $J_1 = \vartheta^2$.

6. For a smoothed stochastic filed expand the *cmd* measure up to $\mathcal{O}(\sigma_0^4)$. Also do for various n. (see https://arxiv.org/abs/2308.03086)

Good luck, Movahed