In the name of God

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OPTIMIZATION METHODS IN PHYSICS

Exercise Set 7

(Due Date: 1400/09/19)

1. Fisher information matrix: In question 1 of exercise 6, you determined the best fit value for H. Having mentioned value and according to the covariance matrix for observed data (now we consider that this matrix plays the role of covariance for systematic errors), evaluate the relative likelihood for H. Compare your result with question 1 of exercise 6.

2. Fisher information matrix: In question 2 of exercise 6, you computed the best fit values for $\{\Theta\} \equiv \{\Omega_m, \Omega_\lambda, \omega, H_0\}$. Now taking into account the COV.txt as the covariance of systematic errors, determine the contours for 1σ, 2σ and 3σ for each pairs of parameter in addition to one-dimensional likelihood analysis.

3. Fisher matrix for a differential equation: Suppose that the theoretical prediction is given by following equation:

$$\frac{d^2G}{d\ln^2 x} + \frac{3}{2}\left(\frac{1}{3} + \frac{\Omega_k(x)}{2} - w(x)\Omega_\lambda(x)\right)\frac{dG}{d\ln x} - \frac{3}{2}\Omega_m(x)G = 0$$

here $\Omega_k(x) = [1 - (\Omega_m + \Omega_\lambda)]/x^2$, $\Omega_m(x) = \frac{\Omega_m x^{-3}}{[\Omega_m x^{-3}+\Omega_\lambda x^{-3(1+w(x))}-\Omega_k x^{-2}]}$, $\Omega_\lambda(x) = \frac{\Omega_\lambda x^{-3(1+w(x))}}{[\Omega_m x^{-3}+\Omega_\lambda x^{-3(1+w(x))}-\Omega_k x^{-2}]}$ and $\Omega_k = [1 - (\Omega_m + \Omega_\lambda)]$ and $w(x) = w_0 + w_1 x$. The parameter is $\{\Theta\} \equiv \{\Omega_m = 0.27, \Omega_\lambda = 0.73, H_0 = 70, w_0 = -1, w_1 = 0.1\}$. Now taking into account the x.txt series containing the $\{x\}$ values and COV.txt as the covariance of systematic errors, determine the contours for 1σ, 2σ and 3σ for each pairs of parameter in addition to one-dimensional likelihood analysis.

4. Fisher matrix to asses the interaction between Ar molecules. Suppose that in a thought experiment, we measure the internal energy of Ar for different temperature. The theoretical internal energy per number of particel is given by:

$$E(T) = \frac{3k_BT}{3} + \frac{n}{2}\int u(r)g(r)d^3r$$

here $u(r) = 4\epsilon\left[\left(\frac{\alpha}{r}\right)^a - \left(\frac{\sigma}{r}\right)^b\right]$and the pair correlation is $g(r) = e^{-\beta u(r)+\beta nF(r)}$ with $F = q\beta$, $\beta = k_BT$. If the model parameters are $\{\Theta\} = \{a = 12, b = 6, \epsilon = 0.997kJ/mol, \alpha = 3.40A^\circ, \sigma = 3.45A^\circ, q = 0.001\}$. Suppose that we have one mole ($n = 1$). Now taking into account the x.txt series containing the $\{x\}$ values and COV.txt as the covariance of systematic errors, determine the contours for 1σ, 2σ and 3σ for each pairs of parameter in addition to one-dimensional likelihood analysis. (You can use any other physical assumptions that you think they can be useful)

Good luck, Movahed
