

In the name of God

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OPTIMIZATION METHODS IN PHYSICS

Exercise Set 4

(Due Date: 1400/08/07)

1. Joint PDF:

**A** : For the input data set, compute  $\Delta(\tau) \equiv \int dx dy |p(x, t; y, t + \tau) - p(x, t)p(y, t + \tau)|$  as a function of  $\tau$ . Explain your results.

**B** : For the input data set, compute  $\Delta(\tau) \equiv \int dx_1 dx_3 |p(x_3, t + 2\tau; x_1, t) - \int dx_2 p(x_3, t + 2\tau | x_2, t + \tau) p(x_2, t + \tau | x_1, t) p(x_1, t)|$  as a function of  $\tau$ . Explain your results.

**C** : According to Box-Muller algorithm, generate Gaussian random field with  $\sigma_0^2 = 2$  and  $\langle x \rangle = 3$ . Check your results by fitting a Gaussian function on the computed PDF of your generated data.

2. According to Von-Neumann method, generate a set of random data set in the range  $x \in [1 - 5]$  with PDF as:  $p(x) = \sin(x^2/100) + \frac{1}{\cos(x^3/100)} + x^{-3}$ .

3. PDF transformation: Suppose that in a black box a harmonic oscillator is oscillating and you made a series of snapshots randomly through time. Determine the PDF of the location of the oscillator in the stationary case.

4. Suppose that  $x$  has the Pareto distribution,  $p(x) = \frac{a}{x^{a+1}}$  for  $1 \leq x < \infty$ . Find the probability density function of each of the following random variables:

**A** :  $y = x^2$ .

**B** :  $z = \frac{1}{x}$ .

**C** :  $T = \ln(x)$ .

Good luck, Movahed

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