بسم الله الرحمن الرحیم

جلسه ۲۴/۷/۱۴۰۱

Some Important Key-words

① Macroscopic Coordinates (Quantities)

$T, \bar{V}, P$ — معمولاً تعداد کمی هستند. مستقیماً روشی برای اندازه‌گیری‌شان داریم

معمولاً به حواس ما مربوط می‌شوند

② External variables — متغیرهای خارجی

T, V, P, H, E, N → Independent Parameters

③ Microscopic Coordinates (Quantities)

معمولاً تعداد زیادی هستند

مستقیماً قابل اندازه‌گیری نیستند

$P(\bar{r}_1, \bar{r}_2)$ = Joint Probability function (Two-Point statistics)

④ Extensive and Intensive ← Thermodynamics

Extensive: $\{V, M, L\}$

Intensive: $\{P, \rho, T, \mu\}$

$$dU = đQ + đW$$

$$dU = TdS - PdV + \mu dN$$

$$đQ = TdS$$

$$dW = -Pd\bar{V}$$

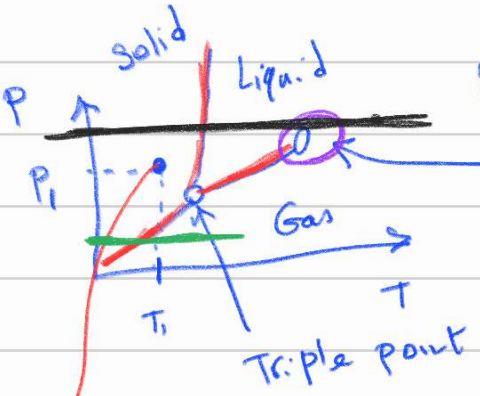
⑤ Order Parameters

پارامتر (کمیت) نظم

☆ Those parameters which are Represented as the Well-Defined response to phase Transition

⑥ Phase and phase Transition

Phase : ☆ If we fix the $\underbrace{\text{external variables}}$, accordingly, some $\underbrace{\text{Makroscopic coordinates}}$ are $\underbrace{\text{uniquely determined}}$, Consequently we have a state quantified by a state (order) Parameter.



$\rho =$ خوش تعریف نیست

— Coexistence phases

$\underbrace{\text{Coexistence phases}}$ First-order phase Transition

☆ In some cases, The uniquness of order Parameter can be eliminated (First-order Phase Transition)

Phase Transition : Changeing in order Parameter as external variables are smoothly varied

Discontinueous phase Transition $\leftarrow$ First order

Continuouse phase Transition $\leftarrow$ Second order

$$\frac{\partial^n F}{\partial K^n}$$

Coupling Constant $\rightarrow H, J, \ldots$

$$\mathcal{H} = \sum K_n \, \Theta_n$$

Ex 1: Paramagnetism : Classical approach

$$\mathcal{H}_i^{\text{mag.}} = -\vec{\mu}_i \cdot \vec{H}_{ext} \qquad \mathcal{H}_N = \sum_{i=1}^{N} \mathcal{H}_i$$

$$\langle \vec{\mu} \rangle = ? \qquad M = N \langle \vec{\mu} \rangle \quad , \quad \vec{H}_{ext} = H_{ext} \, \hat{K}$$

ensemble

متوسط در نقطی هر ذره

مغناطش کل

Canonical Ensemble

$$\langle \vec{\mu} \rangle = \int d\Gamma \; \vec{\mu}(\Gamma) \, \rho(\Gamma)$$

$$\rho = \frac{e^{-\beta \mathcal{H}}}{Z}$$

$$\mathcal{H} = \mathcal{H}_{Kinetic} + \underbrace{\mathcal{H}_{magnetization}} + \mathcal{H}_{Rotation} + \mathcal{H}_{Vibration} + \mathcal{H}_{Internal}$$

$$\downarrow$$

$$F_{mag} \longrightarrow C_H \longrightarrow \chi_H$$

$$C_V = C_V^{Kin} + C_H + C_{vib} + C_{Rot} +$$

$$\downarrow$$

$$\langle \vec{\mu} \rangle = ? \longrightarrow \rho_{mag} \longrightarrow Z_{mag}$$

$$Z_{mag}(T, H, N) = \int d\Omega_1 \, d\Omega_2 \, \ldots \, d\Omega_N \, e^{-\beta \mathcal{H}_{mag}}$$

$$= \int d\Omega_1 \ldots d\Omega_N \, e^{-\beta H_{ext} \sum_{i=1}^{N} \mu_i \cos\theta_i}$$

$$= \int d\varphi_1 \sin\theta_1 d\theta_1 \int d\varphi_2 \sin\theta_2 d\theta_2 \ldots \int d\varphi_N \sin\theta_N d\theta_N$$

$$e^{-\beta H_{ext} \mu_1 \cos\theta_1} \, e^{-\beta H_{ext} \mu_2 \cos\theta_2} \ldots e^{-\beta H_{ext} \mu_N \cos\theta_N}$$

$$Z = \left[\int d\varphi \sin\theta d\theta \, e^{-\beta H_{ext} \mu \cos\theta} \right]^N$$

$$\boxed{Z(T, H, N) = \left[\frac{4\pi}{\beta \mu H_{ext}} \sinh(\beta \mu H_{ext}) \right]^N}$$

$$\vec{\mu} = \mu_x \hat{i} + \mu_y \hat{j} + \mu_z \hat{k} = \mu \sin\theta \cos\varphi \hat{i} + \mu \sin\theta \sin\varphi \hat{j} + \mu \cos\theta \hat{k}$$

$$M = N \langle \mu \rangle = \checkmark$$

$$\langle \vec{\mu} \rangle = \int d\varphi \sin\theta d\theta \, \vec{\mu} \, \frac{e^{-\beta H_{ext} \mu \cos\theta}}{Z(T, H, 1)}$$

$$= \int d\varphi \sin\theta d\theta \, [\overset{\sin\theta\cos\varphi}{\mu_x} \hat{i} + \overset{\sin\theta\sin\varphi}{\mu_y} \hat{j} + \overset{\cos\theta}{\mu_z} \hat{k}] \, \rho(1)$$

$$N \langle \vec{\mu} \rangle = N \langle \vec{\mu_x} \rangle + N \langle \vec{\mu_y} \rangle + N \langle \vec{\mu_z} \rangle$$

$$M = 0 + 0 + N \langle \vec{\mu_z} \rangle = \mu N \left[\coth(\beta \mu H_{ext}) - \frac{1}{\beta \mu H_{ext}} \right]$$

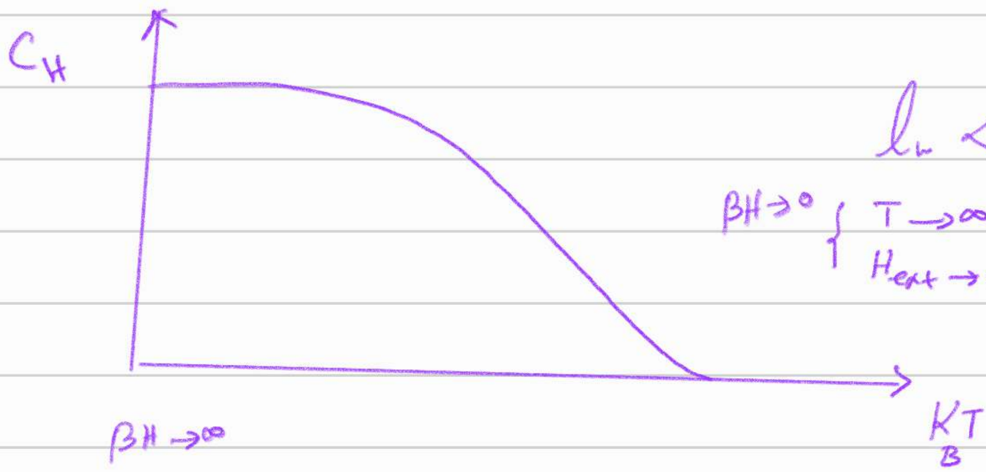
$$\vec{M} = N\langle \vec{\mu} \rangle = \mu N \hat{k}\left[\coth(\underbrace{\beta\mu H_{ext}}) - \frac{1}{\beta\mu H_{ext}} \right]$$

$\beta H_{ext} \to 0$

$H_{ext} \to 0$

$\beta \to 0 \to \frac{1}{K_B T} \to 0 \longrightarrow T \to \infty$

$$C_H = \frac{\partial \langle \mathcal{H} \rangle}{\partial T}\Big|_H = \frac{N K_B}{H}\left[1 - \frac{(\beta\mu H)^2}{\sinh^2(\beta\mu H)} \right]$$



$$\lim \langle \mathcal{H}_{mag} \rangle = 0$$

$\beta H \to 0 \begin{cases} T \to \infty \\ H_{ext} \to 0 \end{cases}$

$\beta H \to \infty$

$T \to 0$

$H \to \infty$

$$\chi_H = \lim_{H \to 0} \frac{\partial M}{\partial H} \simeq \frac{C}{T} \leftarrow \text{Curie's Constant}$$

Exercise 1: Quantum magnetic Moment

$$\vec{\mathcal{H}}_{mag} = \vec{\mu} \cdot \vec{H}_{ext} \qquad \vec{\mu} = (g_l \vec{l} + g_s \vec{S})\mu_B$$

Angular momentum operator

Spin operator

$$\mu_B \equiv \frac{e\hbar}{2mc}$$

$$\vec{j} = \vec{l} + \vec{s}$$

$$\vec{\mu} = g\mu_B \vec{j}$$

g → Total gyromagnetic constant

$$\vec{H}_{ext} = H_{ext}\hat{k}$$

$\mathrm{Cu}\ominus \quad j = 1/2$

$$\mathcal{H} = g\mu_B H_{ext} j_z = g\mu_B H_{ext}\, m \qquad m = -j, -j+1, \dots +j$$

$$\langle \vec{\mu} \rangle = ? \qquad C_H = ? \qquad \chi_H = ?$$

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(7) Symmetry breaking and spontaneous Symmetry breaking and Ergodicity breaking

شکست تقارن ← اغلب گذار فازها با شکست تقارن همراه هستند

گذار فاز: مایع ، جامد

گذار فاز: مایع ← گاز ← هیچ تقارنی شکسته نمی‌شود.

گذار فاز توپولوژیک

تقارن شکسته نمی‌شود $\{$ Topological phase Transit

↓ Cosmic string

چرا اصلا لازم است توجه کنیم به موضوع **شکست تقارن** ← گذار فاز مرتبه دوم

To examine the role of Symmetry in a typical Phase Transition, we imagine a model.

$$\mathcal{H}(H,T,\{S\}) = -\frac{J}{2}\sum_{\langle ij\rangle}^{N} \vec{S}_i\cdot\vec{S}_j - \vec{H}_0\sum_{i}^{N} S_i$$

$H = cts$ external magnetic field

☆ We know that Time-Reversal Symmetry

$$\left.\begin{array}{l} H \longrightarrow -H \\ \{S\} \longrightarrow \{-S\} \end{array}\right\} \Longrightarrow \underbrace{\mathcal{H}(H,T,\{S\}) = \mathcal{H}(-H,T,\{-S\})}$$

☆ $$Z(H,T,J) = \sum_{\{S\}} e^{-\beta\mathcal{H}(H,T,\{S\})}$$

$$Z(-H,T,J) = \sum_{\{S\}} e^{-\beta\mathcal{H}(-H,T,\{S\})}$$

$$= \sum_{\{S\}} e^{-\beta\overbrace{\mathcal{H}(-H,T,\{-S\})}^{\mathcal{H}(H,T,\{S\})}}$$

$$Z(-H,T,J) = \underbrace{\sum e^{-\beta\mathcal{H}(H,T,\{S\})}}_{Z(H,T,J)}$$

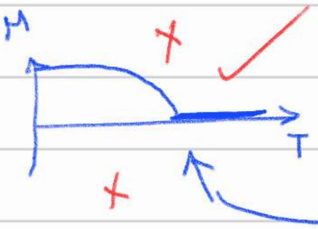
$$F = -K_B T \ln Z(H,T,J) \Longrightarrow \boxed{F(H,T,J) = F(-H,T,J)}$$

$$M = -\frac{\partial F}{\partial H} \longrightarrow M(H) = -\frac{\partial F(H, T, J)}{\partial H}$$

$$= -\frac{\partial F(-H, T, J)}{\partial H}$$

$$M(H) = \frac{\partial F(-H, T, J)}{\partial(-H)} = -M(-H)$$

order parameter $\longleftarrow$ $M(H) = -M(-H)$



if $\cancel{H} \to 0$ $\quad M(0) = -M(0) \longrightarrow \boxed{M = 0}$

چون مغناطش نمی‌تواند بیشتر از صفر باشد مستقل از دما و J

Impossibility of Phase Transion

پس چه چیز اشتباه است ؟!؟

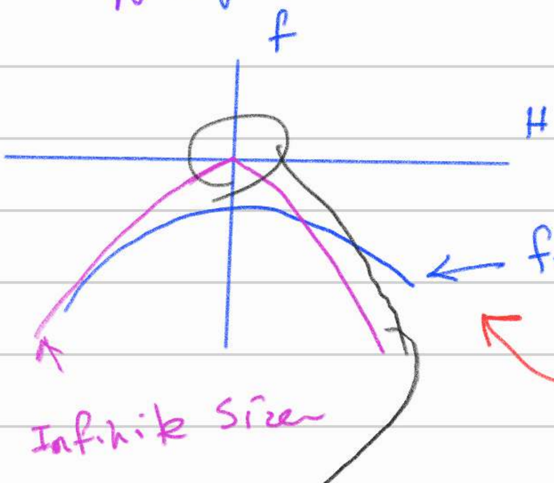
☆ Spontaneous symmetry breaking.

▨ Thermodynamical limit

$M_\Omega(H=0) = 0$ for finite size

$$f = \frac{F(H)}{N \sim V} \longrightarrow f(H) = f(-H) \longrightarrow \text{does not imply } M(H=0) = 0$$

$$\lim_{H \to 0^+} \frac{df}{dH} = \lim_{H \to 0^-} \frac{df}{dH} \quad *$$



finite size

$M(H = 0^+) = -M(H = 0^-)$

$M(0) = -M(0) \rightarrow M = 0$

$$f(H) = f(0) - M|H| + \mathcal{O}(H^{\sigma}) \qquad \sigma > 1$$

$$\frac{\partial f}{\partial H} = \begin{cases} -M + \mathcal{O}(H^{\sigma-1}) & H > 0 \\ +M + \mathcal{O}(H^{\sigma-1}) & H < 0 \end{cases}$$

for $|H| \to 0$

$$M = -\frac{\partial f}{\partial H} = \begin{cases} +M & H > 0 \\ -M & H < 0 \end{cases}$$

$$\left\{ \begin{array}{l} \lim\limits_{N\to\infty} \lim\limits_{H\to 0} \dfrac{1}{N} \dfrac{\partial F}{\partial H} = 0 \\[2em] \lim\limits_{H\to 0} \lim\limits_{N\to\infty} \dfrac{1}{N} \dfrac{\partial F}{\partial H} \neq 0 \end{array} \right.$$