In the name of God

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ADVANCED TOPICS IN MODER COSMOLOGY

Exercise Set 5

(Date Due: 1393/02/30)

1. Determination of phase diagram of Friedmann equation (Dynamical study of cosmological equations). Based on paper arXiv:physics/0108066 compute all fixed points and flux lines around each fixed points.

2. Show that if a general parameter $t = f(s)$ is used to parameterized a straight line in Euclidean space, then the geodesic equation takes the form:

$$\frac{d^2u^i}{dt^2} + \Gamma^i_{jk}\frac{du^i}{dt}\frac{du^k}{dt} = h(s)\frac{du^i}{dt}$$

where $h(s) = -\frac{d^2t}{ds^2}\left(\frac{dt}{ds}\right)^{-2}$.Deduce that this reduces to the simple form $\frac{d^2u^i}{dt^2} + \Gamma^i_{jk}\frac{du^i}{dt}\frac{du^k}{dt} = 0$ if and only if, $t = A(s) + B$, where A and B are constant $A \neq 0$. Here use the signature $(+,-,-,-)$.

3. Energy-momentum conservation in general relativity. Using the general form of $T_{\mu\nu} = \rho u_\mu u_\nu + P\gamma_{\mu\nu} + q_\mu u_\nu + \pi_{\mu\nu}$, derive continuity and Euler equations. Where $\rho = T_{\mu\nu}u^\mu u^\nu$, $P = T_{\mu\nu}\gamma^{\mu\nu}/3$, $q^\mu = -T_{\alpha\beta}u^\alpha\gamma^{\beta\mu}$, $\pi_{\mu\nu}$ is anisotropic pressure tensor, $\gamma_{\mu\nu}$ is 3D Reimannian metric. Use the signature $(-,+,+,+)$.

4. Using Einstein equation as $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$, and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where $g_{\mu\nu} = (c^2, -\frac{a^2(t)}{1-kr^2}, -a^2(t)r^2, -a^2(t)r^2\sin^2(\theta))$. For Ideal flow, derive equation evolution for scale factor. Do the same if the cosmological constant to be added in mentioned equation.

5. State finder: using $r \equiv \frac{d^3a/dt^3}{aH^3}$ and $s \equiv \frac{r-1}{3(q-1/2)}$, determine the corresponding regions for cosmological constant and Chaplygin gas. Here q is deceleration parameter.

6. Cosmic coincidence: Compute and plot $\frac{d\Omega_\Lambda}{d\ln a}$ for various geometry of universe. If the equation of state of dark energy is given by $w_\lambda = w_0a^{-\alpha}$ and $0 \leq \alpha \leq 1$, Investigate cosmic coincidence problem.

Good luck, Movahed